Reinsurance

Abdul H. Rahman; Dick H. Harryvan

bookboon
The eBook company

ABDUL H. RAHMAN AND
DICK H. HARRYVAN

REINSURANCE

Reinsurance
1st edition

ISBN 978-87-403-2234-7

CONTENTS

ABOUT THE AUTHORS

Drs. Dick Harryvan is Vice Chairman of the Supervisory Board of NN Group in the Netherlands an insurance and asset management group operating in 18 countries. He is a former Executive Board Member of ING Group and CEO of ING Direct globally. After obtaining a Masters degree in business economics from Erasmus University, an international career in insurance and retail banking followed. For Nationale Nederlanden Group, he spent nine years in various functions in the US and Canadian business units. The next four years at the international division included amongst others responsibility for setting up new life insurance green fields in Hungary and Czech Republic after the Wall fell. He then moved to ING Bank to develop retail banking internationally for ING Group. In a three-year incubator period, various concepts were tested with responsibility for the ING Direct pilot in Canada. After the initial success in Canada this concept was rolled out to nine countries growing to 24 million customers in the space of 10 years. Since retirement from the ING Executive Board in 2010 several supervisory board positions for ING Group subsidiaries followed. Other positions include membership of the Board of the Dutch automobile association ANWB and partner and investment committee member at Finch Capital private equity fund investing in fintech start-ups.

I dedicate this book my wife Corrie without whom my career would not have been possible.

Rotterdam, The Netherlands, 2018.

Dr. Professor Abdul H. Rahman holds academic degrees that include MSc. (Mathematics); M.A. (Economics); and Ph.D. (Financial Economics). Over his career, he held positions as Full Professor of international economics and finance and Telfer Teaching Fellow at the Telfer School of Management, University of Ottawa, Canada; Associate Dean at what is now the John Molson School of Business, Concordia University in Montreal, Canada; former Academic Director and Founding Director of the Retail Banking Academy, London, United Kingdom and Chairperson of Mathematics and Physics, Dawson College, Montreal, Canada.

He has published in several prestigious refereed academic journals including the Journal of Banking and Finance, Journal of Asset Management, Journal of International Financial Markets, Institutions and Money, Journal of Financial and Quantitative Analysis, Journal of Futures Markets and Review of Economics and Statistics.

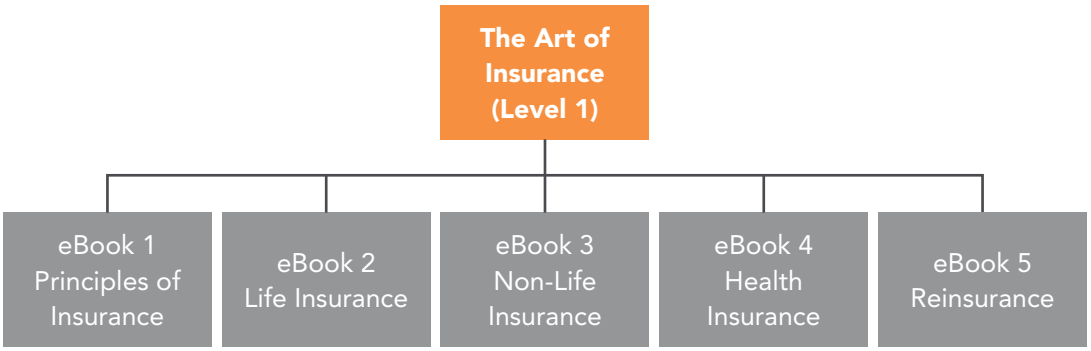
Dr. Rahman has consulted for global financial institutions in banking, asset management and insurance and currently facilitate high-level seminars for senior management professionals in retail and corporate banking, life and non-life insurance and institutional asset management.

I dedicate this eBook to my wife Ruth and daughters Sara and Lisa for their unfailing support of my professional pursuits.

Montreal, Canada, 2018.

PREFACE

The ART OF INSURANCE series is structured according to three levels - I, II and III each with increasing knowledge and understanding of insurance as a business. The architecture of The Art of Insurance (Level I) is schematically summarised as follows:



Each eBook in Level I is developed independently and is self-contained. Hence, the reader is not required to complete the five eBooks sequentially. Consequently, some information presented in the first eBook entitled the *Principles of Insurance* will be repeated so that there is a continuous flow in our presentation.

In addition, our approach focuses on the 'art' rather than the 'science' of insurance where we emphasise principles, concepts and intuition rather than mathematical proofs of complex theorems. While we recognise that the insurance business is founded on a high level of sophistication in probability and statistics, we avoid an overbearing level of jargon and a misplaced reliance on complex equations and formulae.

Chapter 1 discusses the rationale for reinsurance that includes risk-sharing and capital management. We also analyse the structure of reinsurance markets as well as the role of reinsurance brokers in providing reinsurance intermediation. We also discuss the two main forms of reinsurance – facultative and obligatory and highlight their main differences.

Chapter 2 presents a detailed discussion of proportional treaties – quota share and surplus share. We also consider the role of a ceding commission in reimbursing the primary insurer's underwriting expenses as well as its effect on the reinsurer's profit margin. A fixed pre-determined ceding commission ratio as well as its sliding scale counterpart are discussed and well as their incentive effects in the context of expertise transfer from reinsurers to the primary insurer. We explain the role of retention line show to calculate the retention ratio for surplus share reinsurance.

Chapter 3 considers non-proportional treaties – excess of loss, stop loss and catastrophe excess of loss treaties. We highlight the differences between proportional treaties which have *first dollar loss coverage* and non-proportional treaties where the reinsurer pays any additional claim amount above the retention limit up to a maximum value. Excess of loss reinsurance mitigates the primary insurer's vertical exposure arising from high severity claims while CAT XL mitigates its horizontal exposure arising from accumulation of claims.

Chapter 4 considers the pricing of proportional reinsurance which is based on ceded premiums less a ceding commission. There is a possible payment of a profit commission.

Chapter 5 deals with the pricing of non-proportional reinsurance. While the reinsurance premium for proportional treaties is based on ceded premiums, the case for non-proportional reinsurance is not dissimilar to ratemaking principles in non-life insurance. The principle is based on two factors – a measure of the expected gross premium income (EGPI) and a rate which is typically determined by the experience rating method and exposure-rating method. We consider the first case only since it is the most common in practice.

Chapter 6 is a list of references.

1 SETTING THE STAGE: RATIONALE FOR REINSURANCE

1.1 INTRODUCTION

In the first eBook entitled *Principles of Insurance,* it is shown that insurance policies involve a risk- transfer mechanism where a policyholder pays a premium to an insurer in exchange for future contingent payments. Simply put, the insurer will compensate the policyholder who experiences negative economic consequences from a specified insurable event such as fire to residential property, death of a life or diagnosis of a critical illness. For this protection, the policyholder pays a premium to the insurer.

From the insurer's perspective, the risk-transfer mechanism creates a potential for total future payments to exceed total premiums received from policyholders. This is called *underwriting risk.* Consequently, an insurer's exposure has implications for short-term profitability and long-run solvency.

Accordingly, effective risk management which serves to mitigate short-term premium income volatility and to increase the probability of long-term survival of its insurance business, are of paramount importance for an insurance company.

At the core of managing the insurer's exposure to underwriting risk is the *principle of risk pooling*. Specifically, this principle states that, for independent and identically distributed (IID) claims, the insurance company can reduce its exposure by increasing its portfolio size. Simply put, a portfolio of policies (i.e., risks) is diversified through risk pooling.

However, the assumption that policyholders' claims are independent and identically distributed may not necessarily hold.

For example, if the initiation of a claim from one policyholder triggers claims from other policyholders in the portfolio, then risk pooling becomes less effective. This so-called *contagion effect* occurs when large-scale events such as earthquakes, hurricanes and terrorism create significant positive correlation among claims in, for example, non-life and health insurance.

Similarly, terrorism and mass shooting lead to highly correlated claims in life insurance. In actuarial terminology, these catastrophic events create *horizontal exposure* for the insurer. This means that the initiation of a single claim leads to a wider set of correlated claims and hence the possibility of a large aggregate insurer's loss.

Comment

Catastrophe risk creates an accumulation of risk exposure beginning from a single event or occurrence.

In addition, the insurer may face exposure to a few claims with high severity - an example of what is commonly called *vertical exposure.* In statistics, this is called an *outlier problem.* As concluded by Olivier and Pitacco (2010), "when just one sum insured is extremely high if compared to the other sums, the advantage provided by the portfolio size vanishes, so that the riskiness of the portfolio is roughly equal to the riskiness of a portfolio consisting of just one policy".

Comment

In summary, both catastrophe events and large claims render risk pooling less effective as a diversification method.

What are the practical implications for the insurer's exposure to underwriting risk when the IID assumption does not hold? That is, risk pooling is less effective.

The answer to this and related questions and the role of reinsurance in this context are considered in the next section.

1.2 THE ROLE OF REINSURANCE

In this section, we discuss the primary motivation of an insurance company to enter into a reinsurance contract.

Before we provide a formal definition of reinsurance, we first consider the concepts of *systematic risk* and *unsystematic risk*.

As illustrated in Figure 1.1, total risk is typically expressed as a sum of two components – systematic risk and unsystematic risk. Explanations follow this diagram.

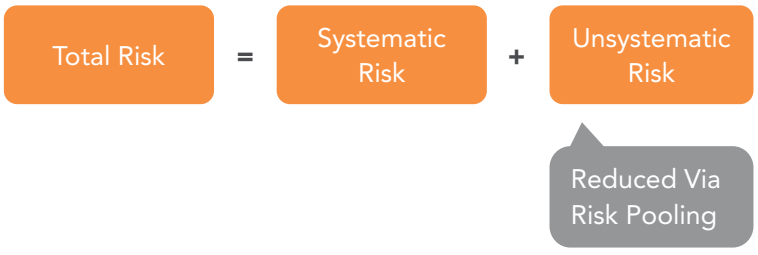


Figure 1.1 Components of Total Risk

Systematic risk is derived from common risk drivers that arise from the overall market or population. An example of systematic risk in life insurance, health insurance and pensions is longevity risk. For example, Jones (2013) in his paper, *Longevity and Reinsurance* states, "longevity risk is the risk of *populations* living longer than expected - for example, through medical advances or declining health risks such as smoking. It is a global challenge driven by the ongoing substantial increases to post-retirement life expectancy and is systematic in nature".

Importantly, systematic risk is not diversifiable. This risk is based on overall market factors and for this reason, it is called *market risk* in investment finance. As emphasised by Bernard (2013*),* "insurance activities cannot be solely based on pooling arguments as issued policies share common risk drivers which can be hard to diversify".

Simply put, pooling of risk based on the law of large numbers is ineffective as a risk-mitigate tool when common risk factors are persistent in society.

The other component of total risk that is depicted in Figure 1.1 is unsystematic risk. This risk is *specific* to an insurer's portfolio of policies. It varies from one insurer to another, depending on their respective internal business model. Hence, it is uncorrelated with systematic risk drivers that determine population trends or market movements. This risk is sometimes described as *idiosyncratic* meaning that is it localised within a particular insurance company.

Comment

Unsystematic risk is specific to the business decisions made by the insurance company and is diversifiable in the sense that it can be reduced by risk pooling.

Caution

We stated above that risk pooling is less effective when the IID assumption does not hold. In this case, *unsystematic risk cannot be reduced completely*. Figure 1.2 shows that systematic risk is independent of portfolio size. Total risk declines and converges towards systematic risk. Observe that some degree of unsystematic risk remains.

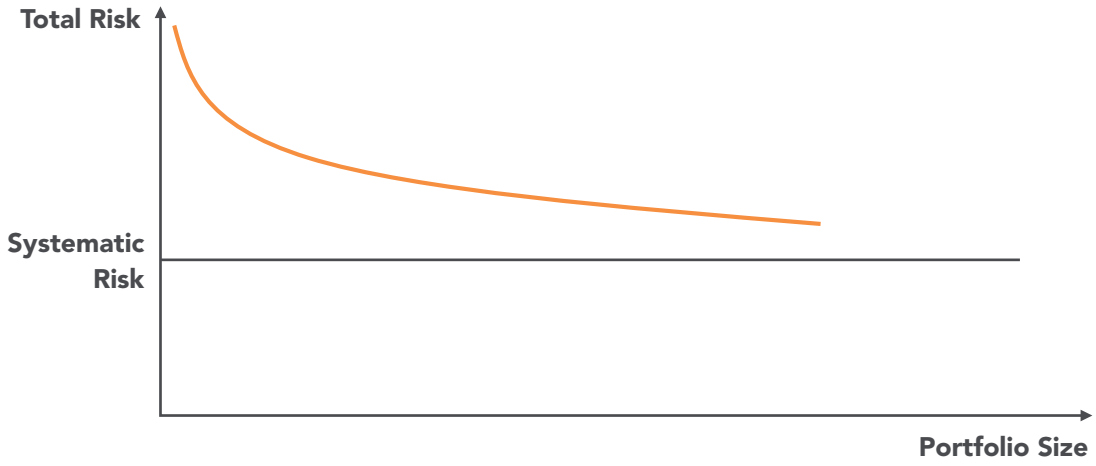


Figure 1.2: Risk Pooling and Total Risk

Since unsystematic risk may not be reduced completely though risk pooling, what options are open to the insurer?

The insurance company can reduce its portfolio risk through risk-sharing contracts in two common ways.

First, the insurer can modify coverage of its written policies through the use of deductibles and policy limits. In this way, the insurer can mitigate its non-life insurance portfolio risk by influencing the share of ground-up losses retained by policyholders. In the third eBook entitled *Non-Life Insurance,* we described a procedure to obtain a fixed-amount deductible corresponding to a pre-determined level of Loss Elimination Ratio (LER) set by senior management.

We also showed how the use of deductibles and policy limits can mitigate risks from anti-selection and moral hazard resulting in a beneficial effect on the non-life insurer's combined ratio.

Second, and the focus of this eBook, the insurer can seek to limit unexpected fluctuations in claims from its portfolio of risks through a risk-mitigating tool called ***reinsurance***.

We now define and discuss the concept of reinsurance.

Definition

Reinsurance is an action by management of an insurance to mitigate underwriting risk. It is based on a contract (called a treaty) between one or group of insurers (*the reinsurer(s)*) and

another insurer (*the primary insurer or cedent*) to indemnify against losses on policies issued by the primary insurer to its policyholders. The losses are shared between the reinsurer and primary insurer who pays a reinsurance premium for this protection.

Simply put, reinsurance is insurance between two professional insurers. It reflects a statement made by Borch (1962) who state that *the participants in this market are insurance companies, and the commodity they trade is risk.*

Comment

a) This definition states that the primary insurer **retains** a portion of its underwriting risk and ***cedes*** the remaining portion to a reinsurer or a group of reinsurers. One can think of the amount of risk retained by the primary insurer as a *deductible,* **D**. We label the amount ceded to the reinsurer, **R** and **S** is the aggregate claim amount for a single risk or portfolio of risks. Then at any point in time *t,* **S = D + R.** In other words, total risk faced by the primary insurer is shared between the primary insurer and the reinsurer.

   The primary insurer's losses arising from policyholders' claims are called *original losses.* Hence, the definition of reinsurance concludes that:

   Original Losses = Retained Losses + Ceded Losses

b) In addition, the primary insurer transfers original premiums paid by policyholders to a reinsurer in exchange for indemnity payments. The reinsurer's (indemnity) payments are due only if the primary insurer faces claims from policyholders. Hence, a reinsurance contract involves three agents – policyholder, primary insurer and reinsurer.

   However, it is noted that reinsurers have a legal relationship only with primary insurers.

   The process underlying the definition of reinsurance is graphically illustrated in Figure 1.3 below:

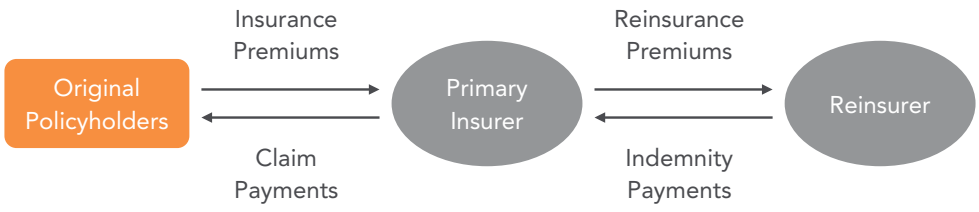


Figure 1.3 Intermediation in Reinsurance

Figure 1.3 highlights a key point – the motivation for a primary insurer to enter into a reinsurance agreement is ***derived*** from its exposure to underwriting risk arising from its relationship with original policyholders. This motivation for reinsurance by primary insurers is explained by Plantin (2006) who asserts that a primary insurer seeks reinsurance for two main reasons.

The first is seen as a risk-management tool – i.e., mitigate short-term fluctuations (i.e., volatility) in losses from policyholders' claims. This is especially important when the underlying assumption of IID claims for risk pooling does not hold as would be the case for catastrophic risk and large claims.

The second reason may be viewed from a capital management perspective – i.e., managing the long-term solvency of the insurance business. Garven and Lamm-Tennant (2003) supports this dual function of reinsurance and state that reinsurance is both a risk management and a financing decision.

Comment

The discussion presented in the previous paragraphs highlights the role of reinsurance in mitigating the primary insurer's underwriting risk. However, the primary insurer accepts counterparty risk – that is, the risk that the reinsurer defaults. Hence, we may conclude that the primary insurer cedes underwriting risk and accepts counterparty risk.

We discuss each of these reasons next.

1.2.1 REINSURANCE AS RISK MITIGATION

The risk-mitigation role of reinsurance is founded on *risk sharing* and pioneered by Borch (1962). As we stated above, risk pooling can diversify *unsystematic risk* to a *certain extent*. However, the potential for catastrophic claims as well as large claims presents constraints to the effectiveness of risk pooling.

This means that the insurer bears some degree of unsystematic risk after risk pooling.

A key point is that by sharing a portion of its underwriting risk with a reinsurer, a primary insurer can reduce or completely eliminate the remaining unsystematic risk, and permit further diversification.

Comment

The risk-sharing benefit of reinsurance is underlined in Neuthor (2013) who state that "through buying reinsurance cover, primary insurers are able to further diversify their remaining unsystematic risk. (page 87)".

Thus comment is reflected in Figure 1.4 below that shows that reinsurance creates further diversification over risk pooling.

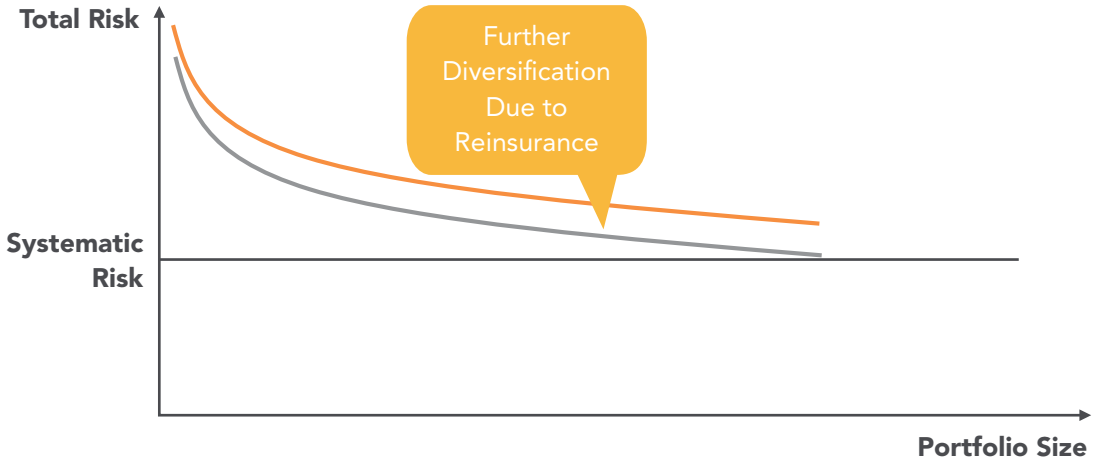


Figure 1.4: Risk Pooling and Total Risk

Comment

The primary insurer obtains insurance from the reinsurer. By this statement we mean that a primary insurer transfers significant underwriting risk to the reinsurer in exchange for a reinsurance premium. Since a reinsurance premium reflects the risk transferred, the primary insurer faces a trade-off decision as follows - paying higher reinsurance premiums for a lower retained risk or retaining more or all of the risk in its portfolio.

If the primary insurer decides to fully retain its underwriting risk, then the portfolio has a relatively higher level of unsystematic risk and hence, higher volatility. This would require a higher level of (equity) capital to support the portfolio risk. The primary insurer may seek to raise more equity capital which is likely to be at a higher cost of capital for the company. This is because equity investors would likely require higher expected rates of return due to the increased volatility of the underwriting portfolio.

Question: Is reinsurance a substitute for equity capital?

We discuss this question by discussing the capital implications of reinsurance from the perspective of the primary insurer.

1.2.2 REINSURANCE AND FINANCIAL LEVERAGE

The primary insurer pays a reinsurance premium for shifting risk to the reinsurer. This premium is a cost incurred by the primary insurer to reduce its volatility in policyholders' claims. This point is seen as follows:

Suppose σ is the standard deviation (i.e., a measure of volatility) of the insurer's portfolio of claims over a specific period of time. After reinsurance, the primary insurer retains 40% of its risk and cedes 60%. The standard deviation of is portfolio claims after reinsurance is equal to $0.40\times \sigma = 40\%$ of the original standard deviation.

Garven and Lamm-Tennant (2003) state that another benefit to the primary reinsurer arises from capital being released due to the reduced volatility of its portfolio of random claim payments. This is explained as follows:

Assume that based on the primary insurer's current portfolio of risks, the current required level of regulatory capital is **K**. After reinsurance, a portion of the primary insurer's underwriting risk is transferred to a reinsurer. Hence, if the resulting required level of regulatory capital is $\hat{\mathbf{K}}$, then $\hat{\mathbf{K}}$ is less than **K.** Thus, the difference $\hat{\mathbf{K}}$ **- K** may be viewed as capital which is released and allows the primary insurer to increase its underwriting capacity.

Comment

Reinsurance creates capital surplus. This increases the underwriting capacity of the primary insurer and facilitates a growth strategy.

What is the implication of the release of capital for the insurance company's financial leverage?

This question was considered by Garven and Lamm-Tennant (2003). Financial leverage provides information on a company's ability to meet its debt obligations. A measure of financial leverage is the ratio of debt to equity. For the same level of debt, the reduction of required regulatory (equity) capital due to reinsurance, results in a decline in the degree of financial leverage. This reduces the *probability of ruin* – a focus of extensive research in modern insurance risk theory.

For this reason, Garven and Lamm-Tennant (2003) assert that insurance companies with high financial leverage are likely to seek higher levels of reinsurance.

Summary

Reinsurance reduces required regulatory capital and as well the overall risk (as measured by financial leverage) of the insurance business. As is commonly-stated: *reinsurance may be viewed as a substitute for equity capital.*

We now discuss the structure of reinsurance markets that are dominated by two global players – Swiss re and Munich Re. In addition, we present the role of a reinsurance broker in facilitating reinsurance treaties.

1.3 GLOBAL REINSURANCE MARKET

Reinsurance markets are simultaneously national and global in their reach in that large reinsurers operate in nearly all geographical regions and supply reinsurance across all products. We recognise that smaller firms focus in specific regions, but the top ten reinsurers have a global focus.

The reinsurance market is oligopolistic in its underlying structure. We explain this assertion in some detail.

First, an oligopoly is a market structure that is dominated, in terms of market share, by a few large firms. No explicit number of firms is required for an industry to be deemed an oligopoly but a general rule of thumb is that the number is between two and ten firms. The key point is that each of these firms is large (e.g., in terms of market share) relative to the overall industry.

A recent report entitled *Global Reinsurance: fit for the future?* by McKinsey & Company (2017) states:

Life reinsurance has higher barriers to entry and is a significantly more consolidated market than P&C reinsurance. The top five life reinsurance companies account for more than 75% of the revenues globally, compared to 40% in non-life reinsurance.

Recent statistics on the global reinsurance market support the view that the reinsurance market is an oligopoly. For example, a recent report by A. M. Best (dated, September 7, 2017) summarises the global and concentrated structure of the reinsurance market that is dominated by two insurers – Swiss Re and Munich Re.

The report states that "ultimately, the takeaway should be that Swiss Re and Munich Re are the reinsurance market's undisputed largest and most influential players. Both have roughly doubled the premium volume of their closest competitors, offer similar capacity across nearly all reinsurance products and geographies and have been the market leaders for well over a decade".

In fact, based on gross written reinsurance premium, the top ten reinsurers have an aggregate market share that is approximately equal to seventy per cent of total market share of the top fifty firms. Even within the top ten reinsurers, Swiss Re and Munich Re dominate. Using statistics reported in 2016 by A.M. Best for gross written reinsurance premium, we calculate that the combined market share of Munich Re and Swiss Re was 44.21% (US$ 69,225 million) of the total for the top ten reinsurers (US $156,595 million).

Comment

The above discussion implies that reinsurers have diversified portfolios across geographical regions and countries so that large claims from a specific insurer in one location are likely instances of unsystematic risk.

Finally, gross written reinsurance premium for non-life insurance is a dominant share of the total premium. For example, while the top two reinsurers have gross written reinsurance premium of US$69,225, a total of US$40,625 or 58.68% is derived from the non-life insurance business.

We now discuss the internal dynamics of the reinsurance market by considering the role of intermediating brokers.

1.3.1 REINSURANCE BROKERS

Reinsurance is purchased by a primary insurer either *directly* from a reinsurer or though a *reinsurance broker.* Reinsurers who are direct writers typically have employees with the required internal expertise and experience supported by adequate resources to underwrite risks transferred from primary insurers.

Figure 1.5 illustrates the direct relationship between a primary insurer and a reinsurer.

Figure 1.5: Direct Writer

We now describe the reinsurance broker channel.

The reinsurance broker channel is dominated by three companies – Aon Benfield, Guy Carpenter and Willis Re – with about seventy-five per cent of the total market share. Hence, the oligopolistic structure of the reinsurance market is also present in the reinsurance broker channel.

The intermediary function of a reinsurance broker typically includes the following:

- Acting on behalf of the primary insurer, the reinsurance broker negotiates a contract (i.e., treaty) between the primary insurer and one or more reinsurers. For this and other services provided, the reinsurance broker receives a fee. The commission paid to the reinsurance broker is almost always paid by the reinsurer.

The following diagram (Figure 1.5) illustrates a typical model of intermediation in reinsurance markets.

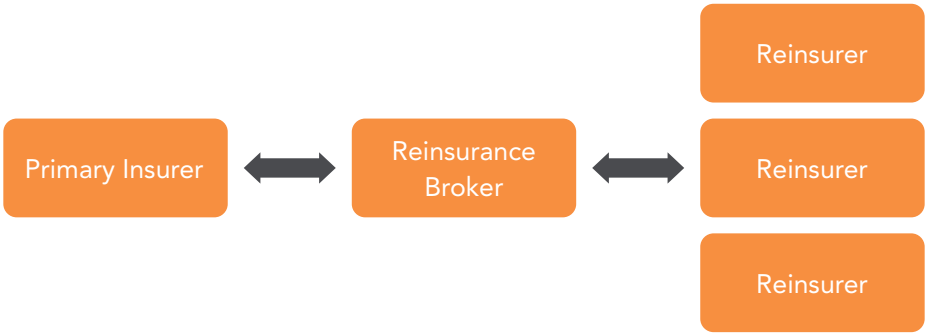


Figure 1.5: Intermediation via a Reinsurance Broker

Comment

The reinsurance broker:

- usually has access to both domestic and international reinsurance markets. This facilitates the successful placement of offers from the primary insurer.
- is not just a middle man but performs more like a market maker.
- assists the primary insurer in the preparation of reinsurance market proposals, prepares contract documentation and advises on underwriting of proposed ceded risk.

Intermediary Clause

This clause is part of the reinsurance contract that identifies the reinsurance broker involved in negotiating the contract, communicating information and facilitating payments between the primary insurer and the reinsurer. The reinsurance broker acts as a conduit facilitating the flow of payments between the cedent and reinsurer. The intermediary clause provides that payments from the cedent to the intermediary broker are deemed to be payments to the reinsurer. Payments from the reinsurer to the intermediary are deemed to be payments to the reinsured only to the extent that such payments are actually received by the cedent.

We conclude this chapter with a discussion of the legal forms of reinsurance.

1.4 FORMS OF REINSURANCE – FACULTATIVE OR OBLIGATORY

A fundamental principle underlying the structure of reinsurance contracts is whether the ceding of risk from the primary insurer or the acceptance of the ceded risk by the reinsurer is *voluntary or obligatory.*

The case of voluntary ceding and/or acceptance of risk is called ***facultative reinsurance***. Facultative reinsurance is transacted between the primary insurer and the reinsurer for a single risk or a package of risks. For example, the reinsurance contract may stipulate a specified risk at a specified location (e.g., losses from fire at a large commercial real estate over the next year).

The decision process involved in placing facultative reinsurance through a reinsurance broker is generally described as follows:

- The primary insurer selects and assesses a specified risk; it may decide to fully retain this risk in its portfolio or retain a portion of the risk and cede the remainder to a reinsurer.
- If the primary insurer decides to share the risk with a reinsurer, then it employs a reinsurance broker to seek a reinsurance solution. This solution will identify a potential reinsurer and a proposed *cession ratio (e.g., 60%)*. The cession ratio is the percentage of risk transferred to the reinsurer.
- The potential reinsurer assesses the terms and conditions proposed by the reinsurance broker and decides to accept or reject the offer.
- If the offer is accepted, a reinsurance contract is signed.

Summary (adapted from examination ST7 (September 2017) sponsored by the Institute and Faculty of Actuaries)

"Facultative reinsurance is an arrangement covering a single risk or a package of risks. There is no obligation for the ceding company to offer the business, nor is the insurer obliged to accept it.
Each case is considered on its own merits and the reinsurer is free to quote whatever terms and conditions it sees fit to impose for that risk."

The other form of reinsurance is called ***obligatory reinsurance*** (also called *treaty reinsurance)***.***

Obligatory reinsurance is based on a well-defined portfolio or block of risks for a period of one-year. (e.g., all automobile accidents in a specified geographical region over a specified period of time (e.g., a year). This portfolio of risks is called a *reinsured portfolio.*

In a negotiation stage, typically conducted by a reinsurance broker, a reinsured portfolio of risks is obtained that meet a set of predetermined conditions required by both the primary insurer and reinsurer. Upon agreement, both parties are obligated to accept the terms and conditions of the treaty.

There are inherent risks in a contract based on *automatic acceptance*. The main reason is that the reinsurer is obligated to accept all risks in the reinsured portfolio while actual insurance policies are issued after the effective reinsurance contract date.

As stated by Neuthor (2013), a clear and unambiguous definition of the reinsured portfolio is required since "the primary insurer starts issuing policies specified by the reinsured portfolio after reinsurance contract inception date".

Comment

Obligatory reinsurance requires an unambiguously-defined reinsured portfolio so as to align and enhance the interests of the reinsurer and the primary insurer. Otherwise, the reinsurer has an exposure to risks that were not priced or not correctly priced in the pre-negotiated stage.

This concludes chapter 1. We now describe and analyse the main characteristics of the main types of reinsurance. Proportional reinsurance is considered in chapter 2 and chapter 3 focuses on non-proportional (or excess of loss) reinsurance.

2 TYPES OF REINSURANCE (PART I): PROPORTIONAL REINSURANCE

2.1 INTRODUCTION

Chapter 1 describes the fundamental characteristics of both facultative and obligatory reinsurance treaties. Both of these forms of reinsurance may be classified as proportional (also called *pro rata*) and non-proportional (also called *excess of loss (XL)*).

An important distinction is that proportional reinsurance involves the sharing of risk between the primary insurer and reinsurer based on *sum insured*. Sum insured represents that risk exposure of the primary insurer in its relationship with original policyholders.

In the case of non-proportional treaties, claims are shared where the primary insurer's liability is capped at a pre-determined level. This cap is called a retention limit. The following diagram summarises this classification:

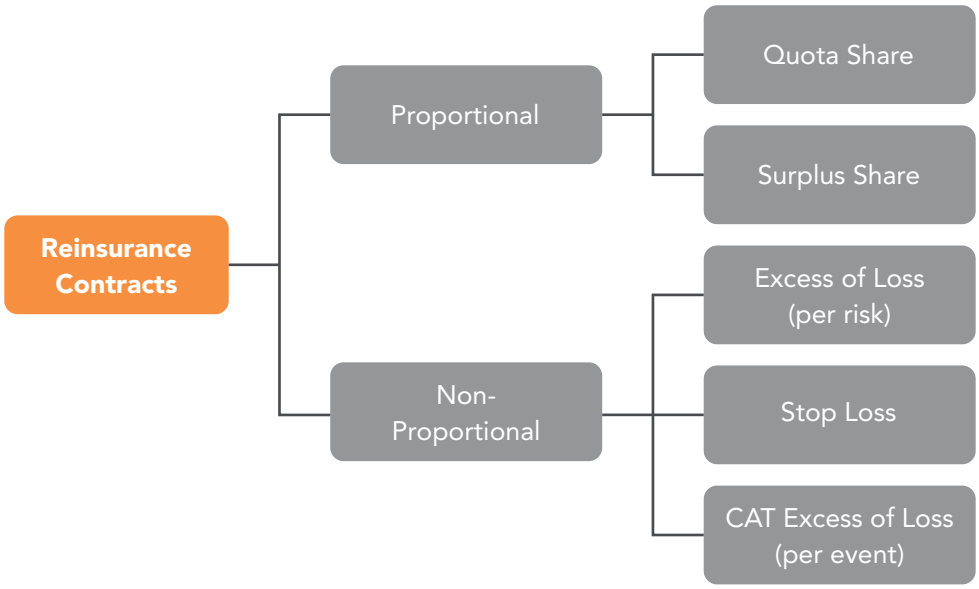


Figure 2.1: Classification of Reinsurance Contracts

Figure 2.1 shows that reinsurance contracts may be classified as proportional or non-proportional. Non-proportional reinsurance is used extensively in P&C reinsurance but is less common in life and health insurance where proportional reinsurance continues to dominate. (Source: International Actuarial Association Risk Book, Chapter 6 by Eves *et al* (2015).

As we stated above, for proportional reinsurance contracts both primary insurer and reinsurer share the sum insured (SI) according to a sharing ratio, $\boldsymbol{\alpha}$ where $0 < \alpha < 1$.

To illustrate, consider a one-year fire insurance policy issued by an insurer with a sum insured equal to $1,000,000. This represents the risk exposure for the insurer in case of fire. It is called the Gross Sum Insured. If the insurer purchases a one-year proportional reinsurance treaty with a retention ratio of 40%, then the risks are shared as follows: 40% of the sum insured is retained by the primary insurer and 60% is transferred (i.e., ceded) to the reinsurer.

Note: (As explained later, the term *gross* means 'before reinsurance' and the term 'net' means 'after reinsurance'.)

Formally:

$$\alpha = \frac{\text{Retained SI}}{\text{Gross SI}}; \quad 1-\alpha = \frac{\text{Ceded SI}}{\text{Gross SI}} \tag{2.1a}$$

Comment

The ratio α is the share of SI retained by the primary insurer. It is called a *retention ratio*; similarly, $1-\alpha$ = share of SI ceded to the reinsurer. It is called a *cession ratio***.**

Once the retention ratio is determined, ***then*** the sharing of premiums and claims between the primary insurer and reinsurer is determined in the same proportion as the sharing of the sum insured.

Simply put, the ratio by which the sum insured (i.e., liability) is shared *determines* the sharing of premiums and claims.

Formally,

$$\begin{aligned} \alpha &= \frac{\text{Retained Premiums}}{\text{GWP}} = \frac{\text{Retained Claims}}{\text{Total Claims}}, \\ 1-\alpha &= \frac{\text{Ceded Premiums}}{\text{GWP}} = \frac{\text{Ceded Claims}}{\text{Total Claims}} \end{aligned} \tag{2.1b}$$

Definition

Gross premium (GP) refers to written premium by the primary insurer ***before*** reinsurance while *net premium (NP)* takes into account the effect of reinsurance. That is, NP refers to written premium that is *retained* through a reinsurance contract.

Comment

The equations in (2.1a and 2.1 b) show that in the case of proportional reinsurance, claims are shared from the first dollar. This is sometimes called *first loss dollar coverage.*

Technical Note

Reinsurance treaties are typically based on two underwriting conventions. The first is called *loss-occurring basis (LOB)* which means that only losses occurring during the reinsurance contract period are covered. For example, if the contract period is 1/1/ 2018 to 1/1/2019, then losses on 1/2/2019 are not covered although the policy was written during the contract period. In this case, *earned premium is considered.*

The second is called *risk-attaching basis (RAB)* which means that only policies incepted (issued or renewed) during the reinsurance period contract period are covered. RAB is also called *policies-attaching basis.* For example, if the contract period is 1/1/ 2018 to 1/1/2019, then losses on 1/2/2019 are covered if this policy was written during the period 1/1/2018 and 1/1/2019. In this case, *written premium is considered.*

Since reinsurance contracts are negotiated between the primary insurer and the reinsurer (possibly through a reinsurance broker), then either risk-attaching basis or loss-attaching basis may be used.

To recognise that either conventions may be used and hence either written or earned premium is appropriate, we use the term 'premium' in this eBook. Only when the underwriting convention is specified, will the specific nature of the premium be recognised.

Two common types of proportional reinsurance are *quota share reinsurance* and *surplus share reinsurance*. There is an important distinction between these types of proportional reinsurance. This is as follows:

a) For quota share reinsurance, the retention ratios (and hence cession ratios) are the same for all risks;
b) For surplus share reinsurance, the retention ratios (and hence cession ratios) vary by reinsured risk.

Details are provided in sections (2.2) and (2.3) below.

2.2 QUOTA SHARE (QS) REINSURANCE

In the case of quota share reinsurance, the primary insurer retains a **pre-determined fixed** ratio **(α)** of both the (claim) loss payment and premiums paid by policyholders. This is based on the primary insurer's business covered in the treaty.

(As we shall see in section 2.3, the retention ratio is determined by a formula- based rather than fixed and pre-determined)

For a portfolio of n policies (i.e., risks), the following common notation is adopted:

S = aggregate claims for a specified time period (e.g., annually);
Π = expense-loaded gross premium (GP).

The equations presented in the next table describe the contractual terms of a quota share reinsurance.

Share of premiums retained by a primary insurer	$\alpha \times \Pi$
Share of premiums ceded to the reinsurer	$(1-\alpha) \times \Pi$
Share of claims retained by a primary insurer	$\alpha \times S$
Share of claims ceded to the reinsurer	$(1-\alpha) \times S$
Share of sum insured retained by a primary insurer	$\alpha \times SI$
Share of sum insured ceded to the reinsurer	$(1-\alpha) \times SI$

Example 1

An insurer issued a one-year non-life policy on a residential property with a sum insured of $1,000,000; gross premium (GP) = $5,000. The insurer enters into a quota share reinsurance with a retention ratio of 40%. There is a claim for a payment of $400,000. What are the sharing amounts for GP, claims and sum insured for both the primary insurer and the reinsurer?

Answer

A retention ratio of 40% means that the sum insured is shared as follows: 40% of $1,000,000 = $400,000 is retained by the primary insurer and 60% of $1,000,000 = $600,000 is ceded to the reinsurer. The retention ratio of 40% determines the sharing formula for GP and total claims. Hence, 40% of GP = 0.40 × $5,000 = $2,000 and 40% of total claims = 0.40 × $400,000 = $160,000 are retained by the primary insurer.

Similarly, for a cession ratio of 60%, the reinsurer pays 60% of the claim = 0.60 × $400,000 = $240,000; receives 60% of GWP = 0.60 × $5,000 = $3,000.

The previous discussion indicates that the primary insurer retains a portion (α) of gross premium (GP). But in underwriting insurance policies with its policyholders, the primary insurer incurs 100% of acquisition expenses of policies including agent's commissions, expenses for settlement of claims, premium taxes as well as other administrative expenses. These expenses are included in GP as an expense load.

Does this create a financial problem for the primary insurer?

Let us look at this question from a simple accounting perspective. The following example illustrates the financial disadvantage to the primary insurer. Since this an accounting issue, we use earned premium and separate the expenses.

Earned Premium = $500,000; Claim Payments = $200,000; Expenses = $150,000. Quota share treaty with retention ratio = 40%.

The following table illustrates the financial problem for the *primary insurer which bears all 100% of treaty expenses.*

	Primary Insurer (40% retention ratio)	Reinsurer (60% cession ratio)
Earned Income	200,000	300,000
Claim Payments	80,000	120,000
Expenses	**150,000**	**0**
Profit	-30,000	180,000

The primary insurer makes a loss of $30,000. The joint profit for both partners in the reinsurance treaty is $180,000 - $30,000 = $150,000.

Comment

The reinsurance treaty creates a positive joint profit but the reinsurer makes a profit of 180,000 and the primary insurer makes a loss of 30,000.

This is example of a concept called *Pareto suboptimality* in economics – the group is better off but not all individuals in the group are better off.

The consequence is that there is not incentive for the primary insurer to purchase quota share reinsurance.

To resolve this issue, reinsurance practice creates an expense sharing solution where the reinsurer covers the expenses included in the ceded premiums.

We continue with our example and incorporate an expense-sharing formula as follows:

Since expenses are 30% of premium, then the reinsurer will pay 30% of expenses in ceded premiums and the primary insurer will pay the remaining expenses.

We restate the above table to reflect this sharing of expenses.

	Primary Insurer (40% retention ratio)	Reinsurer (60% cession ratio)
Earned Income	200,000	300,000
Claim Payments	80,000	120,000
Expenses (30% of premium)	**60,000**	**90,000**
Profit	60,000	90,000

Comment

As before the reinsurance treaty makes a joint profit of $150,000. But now each partner (the primary insurer and reinsurer) makes a profit - $60,000 for the primary insurer and $90,000 for the reinsurer.
There is an incentive for each party to join the reinsurance treaty,

The previous discussion is the basis for what is called a ***ceding commission.*** As stated by Albrecher *et al* (2017), "*the reinsurance commission … compensates the cedent for the fact that acquisition costs of policies, costs for the estimation and settlement of claims as well as other administrative costs are carried by the cedent".*

The reinsurer will typically reimburse the primary insurer for these operating expenses by a monetary amount that is typically calculated as a percentage of the ceded premiums. The percentage of premiums received by the reinsurer is called a *ceding commission ratio* and has a direct relationship to the expense loading included in the gross premium.

Simply put, the reinsurer compensates the primary insurer for expenses included in the premiums that were ceded to the reinsurer.

The ceding commission is calculated as follows:

$$\textbf{Ceding Commission} = \textbf{ceding commission ratio} \times \textbf{Ceded Premium}$$

$$= \mathbf{c} \times (\mathbf{1}\text{-}\alpha) \times \Pi \qquad (2.2)$$

In equation (2.2), the *ceding commission ratio* is **c**.

Here is an example that illustrates the calculation of the ceded commission.

Example 2

Quota share reinsurance; cession ratio = 60%; gross premium (GP) = €20 million; ceding commission ratio = 30%. Calculate the monetary value of the ceding commission.

Answer

By intuition, the reinsurer receives 60% of €20 million = €12 million. The ceding commission is 30% of ceded premium (€12 million) = €3.6 million. The primary insurer receives this amount from the reinsurer as a reimbursement for expenses.

Based on (2.2), the ceding commission paid to the primary insurer = 0.30 × 0.60 × €20 million = € 3.6 million.

Comment

Example 2 shows the calculation of the ceding commission using a **fixed and pre-determined** ceding commission ratio. As stated by Schwepcke and Arndt (2004), "the most common form of reinsurance commission is a fixed percentage of the ceded written premiums. The advantages of a fixed commission are the clear terms of the agreement, its ease of handling, and the associated predictability for planning purposes".

Figure 2.2 illustrates the effect of a fixed ceding commission ratio on the cash flows between the primary parties in a quota share reinsurance. Explanations follow.

Figure 2.2: Cash flows for a Quota Share Reinsurance

Comment on Figure 2.2

Before purchasing reinsurance, the insurer receives expense-loaded premiums, **Π** from policyholders and pays claims of value **S.** This is the left-hand side of the diagram representing the tradition insurance business between an insurer and policyholders. Underwriting is in one -direction. The insurer evaluates and prices the pure risk from potential policyholders.

The right hand-side represents the contractual relationship between the reinsurer and the primary insurer, both of which are professional entities. Underwriting is bi-directional and the relationship in terms of risk management is symbiotic. (We describe this relationship in terms of 'reinsurer expertise transfer' later in this section)

The ceded share of total premium is the **cession rate × gross premium** = $\mathbf{(1-\alpha)\times\Pi}$**.**

The primary insurer deducts the amount of the ceding commission from the premium ceded to the reinsurer.

This means that the ceded gross written premium is reduced by the ceding commission resulting in a transfer to the reinsurer of **(1-α) × Π × (1-c).** Finally, the reinsurer pays a share of the claim that is equal to **(1-α) × S.**

Comment

Both the cession ratio and the ceding commission ratio are pre-determined and known at the effective date of the reinsurance treaty. This is an important point when we discuss pricing for proportional reinsurance in chapter 4.

Example 3

Quota share reinsurance; gross premium = $1,000,000; Cession ratio = 60%; ceding commission ratio = 30%. What is the net amount of premiums ceded to the reinsurer?

Answer

Gross Premium (GP) = $1,000,000; Cession ratio = 60% and ceded commission ratio = 30%. Therefore, ceded GP = Cession rate × GP × (1- ceding commission ratio) = 0.60 × $1,000,000 × (1-0.30) = $420,000.

Comment

The ceding commission, once fixed by agreement between the primary insurer and reinsurer, does not change over the term of the reinsurance contract. This is the case even if the primary insurer's claim experience changes during the term of the contract.

An important issue for negotiation between the primary insurer and reinsurer is to determine the ceding commission ratio so that the amount of ceding commission compensates the primary insurer for the operating expenses included in the ceded premiums.

While there are some advanced methods to determine the ceding commission ratio, we consider the simplest case of a fixed ceding commission ratio in the context of the reinsurer's profit margin.

2.2.1 QS REINSURANCE AND FIXED CEDING COMMISSION RATIO

We begin by consider the expected profit margin from the reinsurer's perspective. Here is an equation showing the factors that determine the reinsurer's profit margin derived from the reinsurance treaty.

Expected Reinsurer Profit (ERP) = Ceded Premium
– Expected Claims
– Ceding Commission
– Reinsurer Expenses (2.3)

The equation for expected reinsurer's profit in (2.3) is simplified as follows:

(i) Ceded Premium = $(1-\alpha) \times \Pi$;
(ii) Expected Ceded Claim = $(1- \alpha) \times E(S)$;
(iii) Ceding Commission = $(1- \alpha) \times c \times \Pi$**;**
(iv) Expenses = M.

Dividing both sides of (4.3) by ceded premium, we obtain:

Expected Reinsurer Profit Ratio (ERPR) = 1- Expected Ceded Claim Ratio – Ceding Commission Ratio – Expense Ratio (2.4)

Using the notation above,

ERPR = 1- Expected Claim Ratio – c – m (2.5)

In this case, m = M / Ceded Premium = reinsurer expense ratio

Equation (2.5) states that the expected reinsurer's profit ratio is reduced by three factors: the primary insurer's expected claim ratio, the reinsurer's ceding commission ratio and its expense ratio.

Here is an example illustrating the expected reinsurer profit ratio for a QS treaty based on equation (2.5)

Example 4

QS reinsurance; cession ratio = 60%; GP = 10 million; Expected total claims = 5 million; ceding commission = 1.8 million; reinsurer expenses = 0.6 million. Calculate the expected reinsurer profit ratio.

Answer

Ceded premium = 60% of 10 million = 6 million; Expected ceded claims = 60% of 5 million = 3 million. Hence expected claim ratio = 3 million / 6 million = 0.50. Reinsurer expenses = 0.6 million, so that the reinsurer expense ratio = 0.6 million / 6 million = 0.10. The ceding commission ratio = 1.8 million / 6 million = 0.30.

The expected reinsurer profit ratio = 1 – 0.50 – 0.30 – 0.10 = 0.10 = 10%.

Equation (2.5) has an important implication, namely, using the ceding commission ratio as an incentive for the primary insurer to reduce the expected claims ratio.

We stated above that many experts assert that the ceding commission reimburses the primary insurer for expenses included in ceded premiums. However, other experts suggest that it is in the interest of the reinsurer to increase the ceding commission ratio as an incentive for the primary insurer to conduct effective underwriting and thereby reduce the expected claims ratio.

Here is an example illustrating this incentive mechanism

Example 5

Referring to the data provided example 4, we assume that the reinsurer increases the ceding commission by one percentage point if the claims ratio (currently equal to 50%) declines by two percentage points. What is the effect on the reinsurer's expected profit ratio if the actual claims ratio is 46%?

Answer

As shown in Example 4, the expected reinsurer profit ratio before the implementation of the incentive mechanism is 10%. The claims ratio is 46% and ceding commission ratio is increased by two percentage points to 32%.

The expected reinsurer profit ratio = 1 – 0.46 – 0.32 – 0.10 = 0.12 = 12%.

The incentive mechanism benefits both parties to the reinsurance treaty in terms of lower claims ratio and profitability.

Comment

Alternatively, the lowering of the claims ratio is facilitated through the transfer of expertise from the reinsurer to the primary insurer in the areas of underwriting, claims management and new product development. This transfer of expertise is emphasised in, for example, the CA3 examination Paper 2 (February 17, 2017) which is sponsored by the Institute and Faculty of Actuaries, where the relationship with a reinsurer brings expertise to the primary insurer in the areas of:

a) Underwriting where, for example, it becomes particularly difficult to estimate expected future lifetimes of life because of complicated medical history or risky occupations or hobbies; the reinsurer has a vested interest and offers suitable underwriting expertise.
b) Claims management where additional expertise is provided to assess difficult claims (e.g., critical illness).
c) Product development where forward-looking design can increase the probability of market acceptance, reduced lapse and improved profitability. Reinsurers, because of their reach can offer innovative solutions for new product design that a local primary insurer may not be cognizant of.

This transfer of expertise serves to increase the efficiency of claims management (and hence lower expected claims ratio) and the efficiency of operations (and hence lower expense loading, γ). By choosing a ceding commission ratio that is equal to the expense loading, both the ceding commission ratio and the expected ceding claims ratio are likely to decline.

Comment

The lesson here is clear: develop and enhance the mutually-beneficial relationship between a primary insurer and reinsurer(s). Focus on the transfer of reinsurer's expertise over an increase of the ceding commission ratio as an incentive for the primary insurer to undertake efficiency investments.

We now introduce the concept of a *sliding scale ceding commission ratio* which is intended to complement the incentive strategy described above.

2.2.2 SLIDING SCALE CEDING COMMISSION RATIO

In the case of a sliding scale ceding commission ratio, the reinsurance contract typically includes two sets of information at the treaty effective date. These are:

a) a minimum and a maximum value for the commission rate and,
b) a description of the sliding scale.

Based on this information, a *provisional* ceding commission percentage is typically derived as the *midpoint* of the interval defined by the maximum and minimum ceding commission ratio. A sliding scale is typically dependent on the level of the primary insurer's claims ratio at end of the reinsurance contract (e.g. one year).

The underlying principle of a sliding scale is as follows:

If the primary insurer's claims ratio is declines over the term of the reinsurance treaty, then the reinsurer increases its ceding commission ratio from the initial provisional ratio. This is a case of a reward for the primary insurer which is bounded by the maximum ceding commission ratio.

In practice, there is no reduction of ceding commission ratio in event of higher claims than expected. This is because the primary insurer would face a double hit of higher claims and lower ceding commission ratio. Rather in the case of higher than expected claims and a loss for reinsurer, the reinsurance premium will be increased at renewal.

Here is an example that illustrates the concept of a sliding scale commission ratio.

Example 5

A non-life insurer enters into a one-year quota share reinsurance with a cession ratio of 60%. The GP = € 20 million and current claim ratio = 70%; provisional ceding commission ratio = 30%

Ceding commission ratios are based on a sliding scale as follows:

There is an increase of one-half percentage point in the provisional commission ratio for each percentage point decrease in the current claim ratio.

The maximum and minimum ceding commission ratios are 35% and 25 % respectively.

a) What is the resulting ceding commission ratio if, upon the termination of the reinsurance, the non-life primary insurer achieved a claims ratio of 65% or five percentage points below the current value of 70%?
b) What is the ceded GP?

Answer

a) The claims ratio declined by 5% and the sliding scale ceding commission ratio is increased by 2.5% from 30% to 32.5%. This is below the maximum value of 35% and hence, the final value of the ceding commission rate is 32.5%.
b) The ceded GP based on the ceded commission ratio in a) is obtained as follows: Ceded GP = cession ratio × GP× (1- ceding commission ratio) = 0.60 × € 20 million × 0.675 = € 8.10 million

Comment

Although the sliding scale ceding commission has the potential to create an incentive to reduce the claims ratio and possibly improve the reinsurer's profit margin, it should be viewed as a complement to the transfer of reinsurer expertise to the primary insurer.

The next section shows that quota share reinsurance frees up capital which may be used to increase underwriting capacity or to reduce the probability of financial ruin for the insurance company.

2.3 QUOTA SHARE AND IMPLICATIONS FOR THE PROBABILITY OF FINANCIAL RUIN

In our discussion on quota share reinsurance, we note that the sum insured (i.e., primary insurer's liability) is reduced to an absolute level determined by the retention ratio. Intuitively, there is less risk exposure and less regulatory capital is required. Some experts in reinsurance summarise this intuition by stating that quota share reinsurance is a substitute for capital.

We present an intuitive explanation of the role of quota share reinsurance is affecting the company's probability of financial ruin.

To set up the main issue, consider an insurance company has allocated a level capital **K** to support its portfolio of risks. Let **Π** be the single payment of portfolio premium, net of expenses. Hence both **K** and **Π** are known at time 0. We label **Π +K,** the company's capital safety level.

However, the portfolio of risks creates future liabilities in terms of claims, **S.** Clearly S is a random variable.

The main question of this section is as follows:

Does quota share reinsurance lead to a lower probability of ruin compared to the case of no reinsurance?

The answer to this question is provided by Charpentier (2010) who concluded that in the proportional case, reinsurance always lowers the probability of ruin. The proof is quite technical and so we will provide an intuitive explanation in the case of quota share reinsurance. A simple rationale of our explanation is provided in the box below:

Without Quota Share Reinsurance:

The insurer faces the chance of financial ruin if future claim payments is higher than the safety level equal to **Π** + **K.** In addition, if capital drops below required regulatory capital (RRC) the regulator will likely step in and impose restrictions on the business practices of the insurer.

With Quota Share Reinsurance:

For a retention ratio(α), the insurance company (as primary insurer) retains both total premiums equal to $\alpha \times \Pi$ and claim payments equal to $\alpha \times \mathbf{S}$. In this case, less capital is required to support the insurer's portfolio of risks. Clearly, the lower the retention ratio (equivalently, the higher the ceding ratio), less capital is required to support risk and so more capital becomes available.

The box below presents some technical details and intuitive explanations follow.

Box 2.2

Quota Share Reinsurance and Probability of Ruin – A Simplified View

In the case of no reinsurance, senior management wants to minimise the probability that $\mathbf{S} \geq \Pi + \mathbf{K}$. We denote this probability by the symbol, ε. That is, Prob $(\mathbf{S} \geq \Pi + \mathbf{K}) = \varepsilon$.

With quota share reinsurance, the probability of ruin with quota share reinsurance is calculated as follows:

$$\text{Prob}\,(\alpha \mathbf{S} \geq \alpha \Pi + K) =$$

$$\text{Prob}\,(S \geq \Pi + \frac{K}{\alpha}) =$$

$$\text{Prob}\,(S \geq \Pi + K + \frac{1-\alpha}{\alpha} \times K) \qquad (2.7)$$

Comment on Box 2.2

Equation (2.7) is interesting in terms of capital released. Here are some key points.

a) It states that quota share reinsurance frees up capital to the amount of $\frac{1-\alpha}{\alpha} \times K$. Hence the company's safety level is higher, so that the probability of ruin is lower compared to the case of no reinsurance. In case of quota share the RRC will also be lowered proportionally providing a larger buffer before regulators would take appropriate restrictive actions.
b) The higher the cession ratio (1-α), the higher is the amount of freed-up capital. If the company wants to keep the same level of safety hurdle as before, the freed-up capital may be used to increase underwriting capacity and grow the insurance business.

We consider the other main type of proportional reinsurance called surplus share.

2.4 SURPLUS SHARE REINSURANCE

Surplus share reinsurance applies to individual risks. The term 'surplus' in surplus share reinsurance refers to the amount in excess of a certain limit hat is ceded to the reinsurer.

Surplus share reinsurance is common in property insurance and it allows the primary insurer to limit its exposure on a risk to a specific amount. This value is a retention limit, **D** which is called a retained ***line.*** The reinsurer accepts a risk with potential for claim payments over the retention level up to a maximum level called the *capacity of the surplus treaty.* The capacity of a surplus treaty is typically expressed as a *number of lines.*

For example, for a retention level of $25,000 (i.e., one line), a *10-line surplus treaty* means that the reinsurer will provide protection for amounts exceeding $25,000 up to and including 10 × $25,000 = $250,000.

Definition

Total Capacity of a Surplus Treaty = Retention Line + Surplus Capacity.

In the previous example, total capacity = $25,000 + $250,000 = $275,000 (i.e., 11 lines) – one line is retained by the primary insurer and 10 lines are provided as additional cover by the reinsurer.

Important Point

The retained line is used to calculate the primary insurer's retention ratio. The retention ratio is formula-driven. Afterwards, the sharing of premiums and claim payments is similar to quota share treaties. Hence, similar to a quota share treaty, surplus share reinsurance provides *first loss dollar coverage.*

But there is key difference: the retention ratio for QS treaties is fixed and pre-determined; the retention ratio for SS treaties is variable and depends on the risk exposure of the policy covered in the contract.

In practice, surplus share reinsurance is used mostly in property insurance while quota share treaties are more common in property and casualty. (Source: Aon Benfield, 2014).

Calculation of the Retention Ratio for Surplus Share Reinsurance

For surplus reinsurance arrangements, let SI be the sum insured for a specified risk (e.g., a commercial property). The primary insurer's retention share (α) is defined as follows:

$$\alpha = \begin{cases} 1 \text{ if } SI \leq D \\ \dfrac{D}{SI} \text{ if } SI > D \end{cases} \tag{2.8}$$

The retention ratio (α) varies with the retention limit (D) and sum insured (SI) for each policy. Once the retention ratio is calculated, surplus reinsurance operates quite similar to quota share reinsurance.

Here are some examples illustrating the definition of the retention ratio in (2.10). Examples 6 through 9 are adapted from Question 1, Exam CAT 12, April 2005 provided by the Institute and Faculty of Actuaries.

Example 6

Assume that the retention level for a surplus share reinsurance is €100,000. What is the retention ratio if the sum insured is €65,000.

Answer

Since the sum insured (SI) is less than the retention limit D, equation (2.12) concludes that the retention ratio is α = 1 meaning that that the primary insurer retains 100% of any loss from a valid claim.

Comment

Note that the cession ratio is the case where SI ≤ D is 0.

Example 7

Assume that the retention level for a surplus share reinsurance is €100,000 and the sum insured is €120,000. Since SI > D, the retention ratio is α = D/SI = €100,000 / € 120,000 = 5/6. The cession rate is 1/6.

Comment

The cession ratio is equivalently calculated from the expression: $\frac{SI - D}{SI}$. From example 8, the cession ratio = (120,000 – 100,000) / (120,000) = 1/6.

Example 8

Using the information provided in Example 7, what is the reinsurer's share for a claim of €60,000?

Note that the value of the claim is less than the sum insured of € 120,000. This is not unusual since a policyholder in non-life insurance, as opposed to life insurance, typically make claims with values less than the sum insured.

The value of α is equal to 5/6. This means that the primary insurer retains 5/6 of the value of the claim. This is calculated as 5/6 × €60,000 = €50,000; the reinsurer pays (1/6) × €60,000 = €10,000.

Example 9

Assume that the retention level for a surplus share reinsurance is €100,000 and the reinsurer's maximum cover is €400,000. For a sum insured of €550,000, what is the reinsurer's share?

Using (2.12), we obtain α = €100,000 / €550,000 = 0.1818. Since the retention ratio is 18.18%, the equivalent cession ratio is 81.82%.

Before accounting for the reinsurer's maximum cover, the reinsurer's share is 0.8182 * €550,000 = $450,000. Since this exceeds the reinsurer's maximum cover of €400,000, then the reinsurer will pay €400,000, and the primary insurer will pay the remaining €150,000.

Here is an example that illustrates the key aspects of surplus share reinsurance.

Example 10

Surplus reinsurance with retention level = $100,000; Surplus Capacity = 2 lines = $200,000. Sum insured = $ 250,000; GWP = $15,000.

What is the reinsurer's share of a claim of $240,000?

Answer

The primary insurer's retention rate = D/SI = \$100,000 / \$250,000 = 40%. The equivalent cession ratio = 60%. The reinsurer's share of the claim payment is equal to 0.60 × \$240,000 = \$144,000 which is below the surplus capacity of 2 lines or \$200,000. This point is explained in the note below.

Note: The total treaty capacity = retention + surplus capacity = \$100,000 + \$200,000 = \$300,000. This value is larger than the sum insured of \$250,000. Therefore, this surplus treaty has enough capacity to cover the sum insured.

Example 11

For a 6-line surplus-share reinsurance treaty with a retained line of \$150,000, how much will the reinsurer pay for a \$500,000 loss on an insured value of \$750,000?

Solution

The retained line is \$150,000 and the reinsured portion could be up to \$150,000*6 = \$900,000. For an insured value of \$750,000, the reinsured portion will be \$750,000 - \$150,000 = \$600,000, so the proportional share of the reinsurer's involvement in any loss would be \$600,000/\$750,000 = 80%. The reinsurer would thus pay 80% of the \$500,000 loss, or **\$400,000.**

Example 12 (adapted from examination ST 8 (April 2015) sponsored by the Institute and Faculty of Actuaries.

The following table provides data on a risk covered under a surplus share reinsurance treaty

Risk	Sum Insured (\$)	Retention Limit (\$)	Retention Ratio	Original Premium (\$)	Ceded Premium (\$)
1	5,000,000	1,000,000	A	34,800	B

Calculate the figures A and B in the table above.

Answer

A = retention ratio = 1,000,000/5,000,000 = 20%.

B = ceded premium = ceded ratio × $34,800 = 0.80 ×$34,500 = $27,840.

Comment

Determining the appropriate retention level for a surplus treaty (or XL reinsurance considered in the next chapter) can be quite complex. The general principle is that the degree of risk retention is directly related to the insurer's ability to obtain relatively liquid funds. There is a more mathematical approach using risk theory (e.g., Hart, Buchanan and Howe, 1996). However, an informal approach is provided by the International Association of Insurance Supervisors (IAIS) in their publication entitled ICP 19 B: Reinsurance, 1996.

This concludes chapter 2.

3 TYPES OF REINSURANCE (PART II): NON-PROPORTIONAL REINSURANCE

3.1 INTRODUCTION

This chapter describes the main features of non-proportional reinsurance treaties. As noted in chapter 2, for proportional reinsurance, sharing between the primary insurer and reinsurer begins with the *first loss coverage*. This is not the case for non-proportional reinsurance the underlying feature of *non-proportional reinsurance* is a *retention limit* that is similar to a fixed-amount deductible in non-life insurance policies. A retention limit (also called a *priority* or a *retention line*) represents the maximum amount of a valid claim retained by the primary insurer.

The reinsurer pays any additional claim amount above the retention limit up to a maximum value, **C**. This is also called a reinsurer's maximum cover. The reinsurance treaty limit is **D + C**. A claim amount higher than the treaty limit is called a *spillover*. A spillover reverts fully to the primary insurer unless it entered into another reinsurance contract.

Note:

a) The focus of non-proportional reinsurance is the sharing of claim payments.
b) The focus of proportional reinsurance is the sharing of risk exposure (i.e., sum insured)

Comment

In actuarial notation and terminology, we have the following:

a) **D** is called a treaty priority or retention limit or deductible; this is the maximum value retained by the primary insurer.
b) **C** is called a treaty guarantee; it is also called a cover limit.
c) **D+C** is called the treaty ceiling.
(As a reminder, a reinsurance *contract* is also called a reinsurance *treaty*)

The actuarial representation of a non-proportional reinsurance is **C xs D** which means that *C in excess of D.* Hence, 100,000 xs 80,000 means that the retention limit for the primary insurer is 80,000 and 100,000 is the maximum payment by the reinsurer for claims exceeding 80,000.

The following diagram (Figure 3.1) illustrates the structure of a non-proportional reinsurance contract.

We observe that value A for Claim #1 in Figure 3.1 is below the retention limit and so this amount is fully retained by the primary reinsurer.

The value of Claim #2 exceeds the retention limit. In this case, the value of the claim represented by B is retained by the primary insurer and the remaining amount C is paid by the reinsurer.

For claim #3, the primary insurer retains a value of G, the reinsurer pays E and the value F is a spillover and reverts to the primary insurer. The primary insurer pays F fully or shares it in another reinsurance contract.

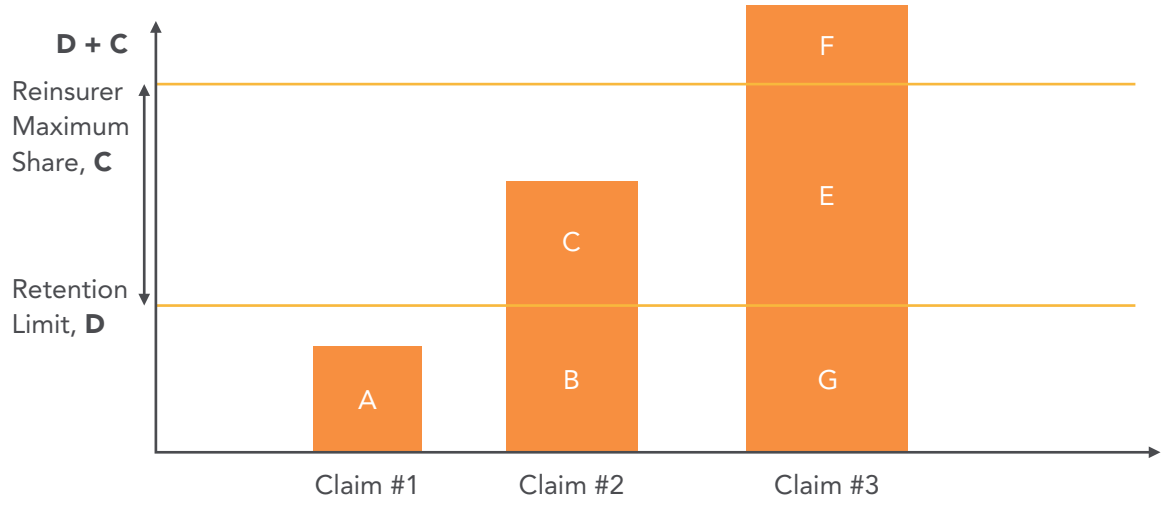


Figure 3.1: Basic Structure of Non-Proportional Reinsurance

Common non-proportional reinsurance contracts are excess of loss (XL) which cover a single risk and called *per-risk reinsurance.* This type of reinsurance is intended to mitigate the primary insurer's vertical exposure for a single risk – that is, for large single claim amounts.

Comment

Excess of Loss reinsurance provides protection against vertical exposure – that is, against high severity claims.

Stop Loss reinsurance is similar in its objective to excess per loss reinsurance except that protection is sought for the primary insurer's *aggregat*e loss over a specific time period (e.g., annually). It is focused on a portfolio of risks.

Catastrophe excess of loss reinsurance (Cat XL) is also called *per-event reinsurance (or per-occurrence reinsurance)* since, as we discussed in chapter 1, a single catastrophic event creates a contagion effect that triggers additional claims by other policyholders. This type of reinsurance is intended to mitigate the primary insurer's horizontal exposure and is common in non-life insurance.

We now consider these three types of non-proportional reinsurance that were also listed in Figure 2.1 of chapter 2.

3.2 EXCESS OF LOSS INSURANCE (PER-RISK REINSURANCE)

Here is an example that highlights the main idea behind this type of reinsurance arrangement. Following this example, we present a formal illustration.

Example 1

Excess of Loss (XL) reinsurance for a single risk; Retention limit = $80,000; Reinsurer's cover = $100,000 which means that the reinsurer will cover a claim payment higher than the retention limit up to a maximum of $100,000. Any amount higher than $180,000 is a spillover that is covered by the primary insurer. This reinsurance is labeled 100,000 xs 80,000.

What is the reinsurer's share of a claim payment of $175,000?

Answer

The retention limit is equal to $80,000. The primary insurer pays this amount. The additional amount of the claim payment is $175,000 - $80,000 = $95,000. The reinsurer pays this amount since it is below the maximum value of $100,000. There is no spillover since the claim payment (after subtracting the retention limit) is below the reinsurer's maximum payment.

Summarising: the primary insurer pays $80,000 and the reinsurer pays $95,000.

Example 2

Based on the information in Example 1, what is the reinsurer's payment if a claim is made for $200,000?

Answer

The retention limit is equal to $80,000. The primary insurer pays this amount. The additional amount of the claim payment is $200,000 - $80,000 = $120,000. This exceeds the maximum payment for which the reinsurer is obligated. The reinsurer pays the maximum amount of $100,000. There is a spillover of $20,000 for which the primary insurer is responsible.

Summarising: the primary insurer pays $80,000 plus a spillover of $20,000 and the reinsurer pays $100,000

Example 3

Based on the information in Example 1, what is the reinsurer's payment if a claim is made for $50,000?

Answer

The retention limit is equal to $80,000. Since the claim payment of $50,000 is below the retention limit, the primary insurer pays the claim in full. There is no payment by the reinsurer.

Summarising: the primary insurer pays $50,000 and the reinsurer pays zero.

These examples are summarised formally by using actuarial terminology for excess of loss reinsurance as follows:

Layer Number	Interval	Cover	Paid By
0	(0, $80,000]	Width of Interval (Retention, **D**)	Primary Insurer
1	($80,000, $180,000]	Width of interval (Cover, **C**)	Reinsurer
2	>$180,000	Spillover	Primary Insurer

Comment

The first layer (0) is represented by the interval (0, $80,000]. This means that any claim payment up to and including $80,000 is retained by the primary insurer. The retention limit is similar to a fixed amount deductible.

The second layer (1) is represented by the interval ($80,000, $180,000]. The width of this interval is **C** = $100,000 which is the maximum cover provided by the reinsurer.

This excess of loss reinsurance is represented by the symbol **C xs D** which is 100,000 xs 80,000. Hence, the first $180,000 of a claim payment is shared between the primary insurer which pays $80,000 and $100,000 for the reinsurer.

Generally, a 3-layer excess of loss reinsurance **(C xs D)** is described as follows:

Layer Number	Interval	Cover	Paid By
0	**(0, D]**	Width of Interval (Retention, **D**)	Primary Insurer
1	**(D, D+C]**	Width of interval (Cover, **C**)	Reinsurer
2	**>D+C**	Spillover	Primary Insurer

Example 4

An excess of loss reinsurance agreement is described as €300,000 xs €100,000. Which one of the following correctly identifies the reinsurer's share of a claim of €250,000?

a) €250,000
b) €100,000
c) €200,000
d) €150,000

The correct answer is d). The primary insurer pays up to the retention limit or the first €100,000 of the claim. The reinsurer pays any additional amount up to €300,000. Since the claim loss is €250,000, the reinsurer will pay the remaining €150,000.

Comment (Comparing Surplus Share with Excess of Loss Reinsurance

For surplus-share reinsurance, the retained line determines the reinsurer's *proportional* share of the risk. For instance, if the retained line is $100,000 on an insured value of $500,000, then the primary insurer will pay 20% of any claim payment and the reinsurer will pay the remaining 80%, *no matter the amount of the loss*. Sharing between both parties begins at the *first dollar of claim amount.*

For an excess of loss reinsurance treaty, the primary insurer covers the first *absolute* amount of a claim (i.e., the retention limit). The reinsurer's obligation to pay comes into effect if the claim payment exceeds the retention limit but is *limited* to a maximum cover. This implies that excess of loss insurance mitigates against high severity claims

We now consider a variation of excess of loss insurance – that is, stop loss reinsurance.

3.3 STOP LOSS REINSURANCE

A variation of excess of loss reinsurance presented in section 3.2 is called **stop loss reinsurance**. While excess of loss reinsurance is typically based on single loss amounts, stop loss reinsurance is related to aggregate loss amounts net of other reinsurance contracts the primary insurer has already negotiated.

While excess of loss reinsurance mitigates against claims of high severity, stop loss reinsurance, since it focuses on the aggregate claim amounts, mitigates against both high frequency and/or high severity claims.

Here is a formal definition of a stop loss reinsurance.

Definition

Let N represent the random number of claims from the insurer's portfolio over a specific time period (e.g., a year). N is called claim frequency. Each claim, $\mathbf{X_i}$ is random amount, let $\mathbf{S = X_1 + X_2 + ... + X_N}$. This means that S is a random sum of random variables and represents the aggregate claims or loss to the insurer.

The following notation is used:

R= retention limit (or priority); L cover limit (or treaty guarantee) and R+L = treaty limit.

Similar in structure to excess of loss insurance, the primary insurer retains an amount equal to a deductible, R. The reinsurer pays that part of S that exceeds R and less than or equal to a limit, L. The limit L is the maximum reinsurer's payment even if S is higher than R + L. The following equation describes the structure of a stop loss reinsurance for the **reinsurer's share** of claims, S.

$$Z = \begin{cases} 0 \text{ if } S \le R \\ S - R \text{ if } R < S \le R + L \\ L \text{ if } S > R + L \end{cases} \tag{3.1}$$

Comment

Similar to excess of loss reinsurance, a stop loss reinsurance is denoted by **L xs R.**

Example 5

Stop Loss Reinsurance; Retention Limit is R= $20,000; Cover Limit is L = $100,000. What is the reinsurer's payment for a claim payment of $60,000?

Answer

The claim payment is S= $60,000 which is in excess of R = $20,000 and below R+L = $120,000. Therefore, the reinsurer's payment is Z = S-R = $60,000 - $20,000 = $40,000.

A variation of equation (3.1) is when the reinsurer's share of claims above the retention level R is further constrained by a percentage that lies between 0% and 100%. Let this percentage be θ which is a co-insurance percentage. Typically, the reinsurer pays the higher co-insurance percentage.

Then (3.1) becomes:

$$Z = \begin{cases} 0 \text{ if } S \le R \\ (1-\theta) \times (S - R) \text{ if } R < S \le R + L \\ (1-\theta) \times L \text{ if } S > R + L \end{cases} \tag{3.2}$$

We explain equation (3.2) by example 4.

Example 6

Stop Loss Reinsurance; Retention Limit = $50,000; Cover Limit = $150,000. Co-insurance percentage is θ = 10%. What is the reinsurer's payment for a claim payment of $100,000?

Answer

The retention limit is R= $50,000 and cover limit is L = $150,000. The co-insurance percentage is θ =10% and the claim payment is S = $100,000 which is less than R+L = $150,000.

Therefore, the reinsurer pays 90% of S – R = 0.90 × ($100,000 - $50,000) = $45,000.

Example 7

Stop Loss Reinsurance; Retention Limit = $50,000; Cover Limit = $150,000. Co-insurance percentage is θ = 10%. What is the reinsurer's payment for a claim payment of $200,000?

Answer

The retention limit is R= $50,000 and cover limit is L = $150,000. The co-insurance percentage is θ =10% and the claim payment is S = $200,000 which is equal to R+L = $200,000. Therefore, the reinsurer pays 90% of L = 0.90 × ($200,000 - $50,000) = $135,000.

We conclude this chapter by describing catastrophe excess of loss reinsurance.

3.4 CAT EXCESS OF LOSS

As we described above, Catastrophe excess of loss (CAT XL) reinsurance which is also called per-event or per occurrence reinsurance, is protection for the primary insurer from horizontal exposure. This arises from a single event that creates contagion effects. Simply put, the event triggers multiple claims within the insurer's portfolio. While the frequency of such an event is low, the accumulated severity can be vey high.

Question

Is the risk of multiple claims on multiple policies mitigated by having surplus share reinsurance? If the answer is yes, then what is the purpose of having CAT XL reinsurance protection?

The answer is that the contagion effect of the catastrophic event across multiple policies has the potential of having a high accumulated amount of *retained losses.* This is because for each surplus share reinsurance, the primary insurer has to cover the retained line for each of the multiple policies making claims at the same time. The probability of ruin may increase beyond a level set by senior management.

Simply put, while surplus share reinsurance does mitigate against large claims, the ***accumulation*** of retained losses from multiple policies can increase the probability of ruin.

Typically, a CAT XL reinsurance comes into effect *after* the primary insurer applies existing reinsurance treaties. *Importantly, it typically covers accumulated retention against catastrophic risks*. (See example 8 below)

Comment

CAT XL treaties provide protection to primary insurers who combine claims incurred in a single event from multiple policies in its portfolio. Simply put, CAT XL provides protection for *accumulation of loss from a single event.*

We explain the dynamics of CAT XL by means of an example.

Example 5
A tornado has affected several high risks in a geographical region. The primary insurer has surplus share reinsurance for each of these risks; each risk has 2 million retention limit and 6-line (12 million) surplus capacity. The primary insurer bought a CAT XL of 8 million xs 3 million. Information on loss amount and sum insured for each risk is provided below.

Risk #	Sum Insured	Loss Amount
1	8 million	8 million
2	5 million	5 million
3	8 million	6 million
4	10 million	9 million

Question

How much of the primary insurer's accumulated retained loss is covered by CAT XL reinsurance treaty?

Answer

Consider the surplus share reinsurance for each of the four risks included in the table above. We calculate the retention ratio for each risk which is obtained as retention ratio divided by the sum insured. For each loss amount, we then calculate the retained loss in each case as the retention ratio multiplied by the loss amount. These calculations are indicated in the table below:

Risk #	Retention Limit	Sum Insured	Retention Ratio (α)	Loss Amount	Loss Retained
1	2 million	8 million	2/8 = 0.25	8 million	0.25 × 8 million = 2 million
2	2 million	5 million	2/5 = 0.40	5 million	0.40 × 5 million = 2 million
3	2 million	8 million	2/8 = 0.25	6 million	0.25 × 6 million = 1.5 million
4	2 million	10 million	2/10 = 0.20	9 million	= 0.20 ×9 million = 1.8 million
					Accumulated Loss Retained = 7.3 million

The table above shows that the accumulated loss retained by the primary insurer is 7.3 million. The CAT XL is 8 million xs 3 million. Hence, the deductible is 3 million. The CAT XL reinsurer pays 4.3 million.

This concludes chapter 3. The next chapter discusses a standard approach for the pricing of proportional reinsurance followed by chapter 5 which considers the pricing of non-proportional reinsurance based on the experience rating method.

4 PRICING OF REINSURANCE TREATIES

4.1 INTRODUCTION

This chapter considers pricing methods for both proportional and non-proportional reinsurance treaties. In terms of risk and expected return, it is justified that reinsurers price treaties to obtain the fair rate of expected return for the risk they bear. The price for a reinsurance treaty is called *reinsurance premium.* The reinsurer's risk from the reinsurance treaty is related to the extent of the risk transferred from the primary insurer.

As we showed in the previous chapters, a reinsurance treaty between a reinsurer and primary insurer is negotiated to reflect, for example, mutually-agreed cession ratios, retention lines, contract terms. Hence the concept of an *average reinsurance premium* is ambiguous.

In addition, as asserted by Clark (2014), there is the notion of a pricing paradox in reinsurance. This paradox states that "if you can precisely price a given contract, the ceding company will not want to buy it".

The implication of this paradox is that demand for reinsurance by an insurer is associated with an economic and financial environment that is inherently volatile. Otherwise, an insurance market that is stable will lead to relatively reliable prediction of future claims ratio. In this case, the insurer will likely retain its portfolio of risks if it has sufficient capital. Therefore, there would be no requirement for risk mitigation through reinsurance.

The implication is that reinsurance premiums obtained by analytical methods must be appropriately adjusted to incorporate the actuary's professional judgement.

The remainder of this chapter comprises the pricing of proportional reinsurance based on a procedure that is relatively straightforward. This is followed by an analysis of the pricing of non-proportional reinsurance based on the experience rating method (ERM) that is the most common of the traditional approaches.

4.2 PRICING OF PROPORTIONAL REINSURANCE

Albrecher *et al* (2017), amongst others describe a procedure to obtain the reinsurance premium for proportional reinsurance. From section 7.3 in their book entitled, *Reinsurance: Statistical and Actuarial Aspects,* we paraphrase this procedure as follows:

Price of a Proportional Reinsurance = ceded share of the original premium ***reduced*** *by a ceding commission and a profit commission (if due).*

The remainder of this section considers the implementation of this procedure.

Using the notation in the first eBook entitled *Principles of Insurance*, we note that gross written premium (GWP) is denoted as follows:

$$\Pi_0 = \big((1+\theta)\times \mathbf{E(S)}\big) + \big(\beta \times \mathbf{E(S)}\big) \tag{4.1}$$

In equation (4.1), θ is a safety loading factor, E(S) is expected claim payments and β is an expense loading factor. The expression in the first bracket is the expected value premium and that in the second bracket is the premium to cover total operating expenses.

With this set-up, we now consider the pricing procedure for a quota share treaty with a retention ratio of α. Therefore, the cession ratio is (1-α) and the share of the gross written premium ceded to the reinsurer is given by the expression:

$$\Pi_0^{\mathbf{R}} = (1-\alpha)\times \Pi_0 \tag{4.2}$$

Simply put, equation (4.2) states that the primary insurer pays a share of premiums from its original policyholders. This share is the cession ratio. This is the price of the QS treaty *after the first step in the pricing process.*

According to the approach described by, for example, Albrecher *et al* (2017), equation (4.2) provides the first component of the reinsurance premium.

Before, we consider the second component, we discuss the implications of (4.2) after substituting equation (4.1). Accordingly, we obtain:

$$\Pi_0^{\mathbf{R}} = (1-\alpha)\times\Big[\big((1+\theta)\times \mathbf{E(S)}\big) + \big(\beta \times \mathbf{E(S)}\big)\Big] \tag{4.3}$$

This is simply explained as follows. Technical details are presented in the next box.

When the primary insurer shares gross written premium with the reinsurer, it also shares, in the same proportion, expected claims, risk load and expense load.

Comment

Equation (4.3) states that the premiums ceded to the reinsurer comprises:

(i) A ceded share of expected claims = $(1-\alpha) \times E(S)$.
(ii) A ceded share of risk loading = $(1-\alpha) \times \theta \times E(S)$.
(iii) A ceded share of operating expenses = $(1-\alpha) \times \beta \times E(S)$

We now consider the next step in the pricing process. This is the case of a ceding commission paid by the reinsurer to the primary insurer. Before we provide the details, we summarise the two steps to this point.

Figure 4.1: Illustration of Reimbursement of Ceding Commission

Figure 4.1 states that the reinsurance premium for a quota share treaty is equal to the ceded share of gross written premiums ***reduced*** by a ceding commission paid by the reinsurer to the primary insurer.

Noting that we provided a relatively detailed presentation on the ceding commission in chapter 2, we consider another aspect as it related to pricing. We consider the case of a *fixed* ceding commission ratio.

4.2.1 FIXED CEDING COMMISSION RATIO

We consider a **fixed** ceding commission ratio, denoted by **c.** As discussed in chapter 2, the reinsurer reimburses the primary insurer for operating expenses which are included in ceded premiums. This reimbursement is called a *ceding commission* and is calculated as follows:

Ceding Commission = ceded commission ratio × Share of Premium Ceded to the Reinsurer

$$= \mathbf{c} \times (1-\alpha) \times \Pi_0 \tag{4.4}$$

Substituting (4.4) into (4.2), we obtain:

$$\Pi_0^R = (1-\alpha)\times(1-c)\times\Pi_0 \tag{4.5}$$

Comment

Equation (4.5) is the formula representing the diagram in Figure 4.1. *It is the reinsurance premium for a quota share treaty. We use the same symbol as we did in equation (4.2) after the first step.*

Example 1

One-year QS reinsurance; cession ratio = 60%; GP = 10 million; ceding commission ratio = 30%. Calculate the reinsurance premium for this quota share treaty.

Answer

By intuition, the primary insurer pays a share of the GP to the reinsurer = 60% of 10 million = 0.60 × 10 million = 6 million. The reinsurer reimburses the primary insurer a value which is 30% of the 6 million = 1.8 million. Hence the net price paid by the primary insurer = 6 million – 1.8 million = 4.2 million.

The final step in our procedure involves a *profit commission* paid by the reinsurer at the end of the term of the treaty if the reinsurer's profit is positive.

4.2.2 PROFIT COMMISSION

The main motivation for a reinsurer's profit commission paid to the primary insurer is to encourage optimal insurance operations on the part of the primary insurer. Clearly, if the primary insurer obtained a lower claims ratio at the end of the reinsurance term, the reinsurer will also profit by having to share a lower level of claim payments.

From the reinsurer's perspective, **actual** profit is obtained as follows:

Profit = Ceded Premium – **Actual** Claim Payments – Ceded Commission – Operating Expenses.

Following Clark (2014, page 13), the reinsurer's **actual** profit is given by the equation:

$$\text{Profit Commission Ratio} = PR \times (1 - ALR - CC\text{- } M) \tag{4.6}$$

where ALR = Actual Loss Ratio;

CC = Ceding Commission Ratio
M = Margin for Expenses and,
PR = Per cent Returned.

The profit commission (monetary value) = profit commission ratio × ceded premium (4.7)

Comment

The profit commission is paid only if, at the end of the reinsurance term, profit is positive.

Example 2

At the end of the term of a quota share reinsurance, the following information is obtained for the reinsurer:

The actual loss ratio is 65%.
The ceding commission is 20%.
The reinsurer's margin for expenses is 4%.
The percent returned from the profit is 50%.

Calculate the profit commission ratio. Based on ceded premium of $12 million, what is the profit commission?

Answer

Profit Commission ratio = 0.50 × (1 – 0.65 – 0.20 – 0.04) = 0.50 × 0.11 = 5.5%.
Profit Commission = 0.055 × $12 million = $0.66 million.

Example 3

At the end of the term of a quota share reinsurance, the following information is obtained for the reinsurer:

The actual loss ratio is 65%.
The ceding commission is 30%.
The reinsurer's margin for expenses is 10%.
The percent returned from the profit is 50%.

Calculate the profit commission ratio.

Answer

Profit Commission Ratio = $0.50 \times (1 - 0.65 - 0.30 - 0.10) = 0.50 \times -0.05 = -2.5\%$
Hence no profit commission is due.

Comment

The previous two examples show that the payment of a profit commission by the reinsurer is uncertain. IT depends on the reinsurer earning a positive profit from the treaty.

Summary

The principle underlying the pricing of proportional reinsurance treaties is illustrated as follows:

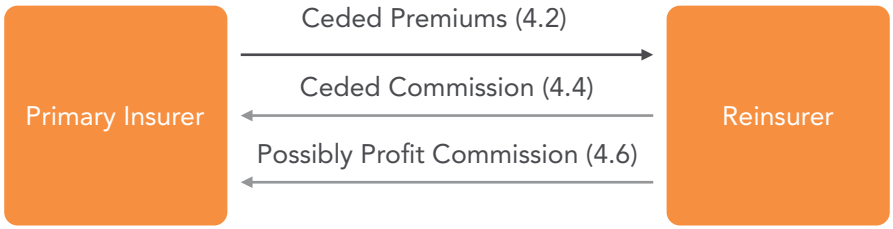


Figure 4.2 Pricing Process for Proportional Reinsurance. Relevant equations are indicated

In Figure 4.2, it is noted that main part of the price of a proportional reinsurance treaty is the ceded share of premiums. The second most important component is the ceding commission. A profit commission is possible.

Comment

One of the challenges facing the actuary is to obtain a reliable and valid estimate of the ceded commission rate. A report by Aon Benfield, one of the dominant companies in the reinsurance broker market suggests two approaches where the ceding factor is calculated to:

a) target the primary insurers' expenses and,
b) deliver a desired expected reinsurer profit margin.

We provide two examples that illustrate these two approaches to obtaining the ceding commission rate.

Example 4

Objective: Calculate the reinsurance premium for a one-year surplus share treaty.

Information provided:

- Sum Insured = 5,000,000
- Retention Limit = 1,000,000
- Gross Premium = 34,800
- Expense loading factor = 25%.
- The average of claims ratio over the last 5 years is 60%. This calculation is based on data for the annual gross premiums and claims (including loss adjustment expenses, LAE) adjusted by the actuary for trends, inflation and other relevant factors.
- The reinsurer's expense ratio is 10%

What is the reinsurance premium for the quota share treaty if the ceded commission ratio is equal to the primary insurer's expense loading factor?

Answer

a) The retention ratio = retention limit / sum insured = 20%. Hence, the cession ratio = 80% and the ceded premiums = 80% of 34,800 = 27,840.

b) The ceding commission = ceding commission ratio × ceded premium = 0.25 × 27,840 = 6,960.
Hence, the reinsurance premium after taking into account the effect of the ceding commission = 27,840 – 6,960 = 20,880.

c) Suppose that at the end of the term of the treaty, the actual claim ratio is 65%. What is the profit commission paid to the insurer for a return percentage of 50%?
By equation (4.6), the Profit Commission Ratio = PR × (1 – ALR – CC- M) = 0.50 × (1-0.65 – 0.25 – 0.10) = 0.50 × 0.00 = 0%.
The reinsurer expected a profit margin of 5% at the start of the surplus share treaty. Since the actual claims ratio is 5% higher at the end of term, the actual profit is reduced by 5% to a final value of zero.
There is no adjustment to the reinsurance premium due to a profit commission.

Example 5

Objective: Calculate the reinsurance premium for a one-year quota share treaty.

Information provided:

- Retention ratio = 40%
- The primary insurer's expense loading factor is 28%.
- Gross Premium = 20 million.
- The average of claims ratio over the last 5 years is 57%. This calculation is based on data for the annual gross premiums and claims (including loss adjustment expenses, LAE) adjusted by the actuary for trends, inflation and other relevant factors.
- The reinsurer's target expected profit is 8%;
- The reinsurer's expense ratio is 6%.

What is the reinsurance premium for the quota share treaty?

Answer

The reinsurance premium is equal to the ceded premium minus a ceding commission.

a) The ceded premium = cession ratio × gross premium = 60% × 20 million = 12 million.
b) The ceding commission ratio is obtained as follows:

Since the target expected reinsurer profit ratio is 8%, we refer to equation (2.5) of chapter 2 which states:

ERPR = 1- Expected Claim Ratio – c – m.

Equivalently, c = 1 – Expected Claim Ratio – m – target ERPR
= 1- 0.57 - .06 -.08 = 0.29 = 29%

Hence, a ceding commission rate of 29% is consistent with an target expected reinsurer profit margin of 8%.

The corresponding ceding commission = 29% of the ceded premium = 0.29 × 12 million = 3.48 million.

The reinsurance premium after taking into account the effect of the ceding commission = 12 million – 3.48 million = 8.52 million.

c) Suppose that at the end of the term of the treaty, the actual claim ratio is 60%. What is the profit commission paid to the insurer for a return percentage of 50%?

By intuition, since the claim ratio is 3% higher, then the reinsurer's actual profit falls from 8% to 5%.

The profit commission ratio = 50% of 5% = 2.5%.

The same result is obtained from (4.6) which states that the Profit Commission Ratio = PR × (1 – ALR – CC- M) = 0.50 × (1-0.60 – 0.29 – 0.06) = 0.50 × 0.05 = 2.5%

Equation (4.7) states the profit commission is 2.5% × 12 million = 0.3 million.

There is an end-of term payment to the primary insurer of 0.3 million.

Comment

Example 4 shows that there is no *definitive market-derived* reinsurance premium for proportional reinsurance. The cession ratio is negotiated; the determination of the ceding commission ratio is not unique and the payment of a profit commission is not guaranteed.

Experts in *forward derivative* contracts will observe similarities since these contracts are tailor-made as well.

This concludes chapter 4.

5 PRICING OF NON-PROPORTIONAL REINSURANCE

5.1 INTRODUCTION

In chapter 3, we show that non-proportional reinsurance allows primary insurers to transfer a significant percentage of large claims (i.e., exceeding a retention limit) for single risks or portfolio of risks to reinsurers. By so doing, primary insurers reduce their regulatory capital requirements for its existing portfolio of risks.

But what is the price charged by the reinsurer for this protection and capital benefit?

In chapter 4, we show that the pricing of proportional reinsurance is based on a principle where the primary insurer pays a premium to the reinsurer that is a proportion of the original premium from policyholders. Adjustments are made to ceded premiums to incorporate a ceding commission and the *possible* payment of a profit commission.

Simply put, for proportional treaties, reinsurance premium depends on primary premiums and the reinsurer pays a ceding commission.

This approach does not apply for the pricing of non-proportional reinsurance. Rather the calculation of a reinsurance premium depends on several variables such as the type of reinsurance treaty (e.g., excess of loss (XL), stop loss or CAT XL); the primary insurer's claims experience history and the structure of the layer that is being rated (i.e., priced).

The general approach to determine an estimate of the reinsurance premium for a non-proportional treaty is similar to the one described in the third eBook entitled *Non-Life Insurance.* This approach states that:

$$\text{Premium} = \text{Rate} \times \text{Number of Units of Exposure} \tag{5.1}$$

A rate is defined as the price of a unit of exposure and the number of units of exposure is a volume measure of risk. Hence, the premium represents the total cost of the risk transferred from the insured to the insurer.

We now discuss the approach (defined in equation (5.1) to determine the reinsurance premium for an XL reinsurance treaty.

5.2 PREMIUM FOR AN XL REINSURANCE TREATY

In the context of no-proportional treaties, the number of units of exposure is called the ***subject premium.***

A *first estimate* of the subject premium is the gross premium (GP) received by the primary insurer from its policyholders. This is an estimate of the volume of risk exposure of the primary insurer in its relationship with policyholders. We label this value as the first estimate of the primary insurer's total risk exposure required to implement (5.1). This is because adjustments to the first estimate of the GP are required. These include:

a) lost premiums from lapse and or cancelled policies which will reduce the primary insurer' risk exposure and,
b) *inuring* for the benefit of the *reinsurer*. The term *inuring* means *acting or serving* for the benefit of someone. We will use this intuitive meaning in our subsequent discussion using the word *inuring.*

We explain this actuarial concept in example 1.

Example 1

A primary insurer has a quota share reinsurance with a 60% cession rate and an excess of loss reinsurance stated as 500,000 xs 100,000. Assume a claim payment of $ 300,000.

Serving for the benefit of the *XL reinsurer*:

In this case, the quota share reinsurer pays 60% of $300,000 = $180,000. For the remaining $120,000 and with respect to the XL treaty, the primary insurer retains $100,000 and the reinsurer pays $20,000.

Comment

In the case where the quota share treaty serves for the benefit of the primary insurer, the XL reinsurer pays $200,000 of the claim of $300,000 and the primary insurer pays $100,000 which is subject to the quota share reinsurance. Hence, the primary insurer pays 40% of $100,000 = $40,000. This is 40.000/300,000 = 13.33% of the claim. The XL reinsurer pays 66.67% and the quota share reinsurer pays $60,000 or 20% of the claim.

Summary

Adjusted Subject Premium = Subject Premium – Adjustments (e.g., return and cancelled premiums, serving for the benefit of the XL reinsurer).

The adjusted subject premium is also called gross net premium income (GNPI).

Example 2

Given the following information:

Subject premium = $100, 800,000; Return and Cancelled premiums = $3,500,000. Serving for the benefit of the XL reinsurer = $30,600,000. Calculate the adjusted subject premium income.

Answer

Adjusted Subject Premium = Subject Premium – Adjustments = $ 100,800,000 – 3,500,000 – 30,600,000 = $66,700,000.

The net step in the process to determine the reinsurance premium is to obtain an estimate of a reinsurance *rate* as shown in (5.2). Theoretically, the applicable rate is forward-looking since it applies to the future reinsurance period.

There are two common approaches to estimate the reinsurance rate. These are:

a) The *experience rating method* that considers the primary insurer's claim experience to predict a claims ratio.
   This is a common approach to obtaining the reinsurance rate.
b) The *exposure rating method* that is based on partitioning the risk portfolio according to risk exposure (i.e., sum insured).

Comment

There are other advanced methods based on stochastic models which are relegated to Level II of the Art of Insurance Series.

The general principle applied to determine an estimate of a reinsurance premium may be illustrated diagrammatically as in Figure 5.1 below. Explanations of the actuarial terminology included in Figure 5.1 follow.

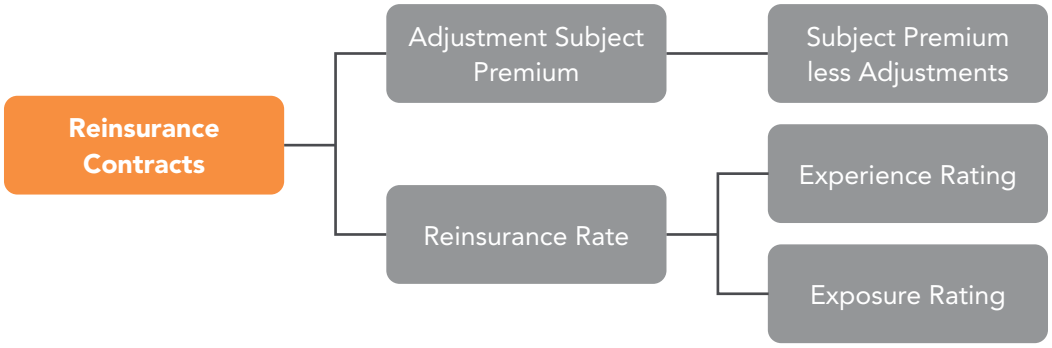


Figure 5.1 Adjusted Subject Premium for Non-Proportional Treaties

The model in Figure 5.1 forms a basis for estimating the reinsurance premium for non-proportional treaties and states that:

$$\textbf{Reinsurance premium} = \textbf{Adjusted Subject Premium} \times \textbf{Reinsurance Rate} \quad (5.2)$$

In the next section we describe the procedure to obtain an estimate of the reinsurance rate based on claims experience (also called the Burning Cost Method).

Note: This section considers only the experience rating method. The other two methods to obtain the reinsurance rate are more advanced and rely on exposure curves and probability distributions. We will consider these method in Level II of the Art of Insurance series.

5.3 EXPERIENCE RATING (ALSO CALLED THE BURNING COST METHOD (BCM))

We illustrate the experience rating method which is based on claims experience by an example.

Example 3

Objective: Estimate the premium rate for an XL reinsurance treaty for a specific layer denoted by C xs D.
For this example, the layer is 100,000 xs 80,000.

First Step:

Obtain information on historical (incurred) losses *from ground-up* (i.e., fgu), which is a common actuarial notation for a selected period, typically 5 years. Clearly, some years will contain several ground-up losses.

Here is a key point: apply the XL layer to the ground-up losses *as* ***if this layer was in effect***. Then calculate the corresponding XL reinsurer's losses.

For example, if an insurance policy for last year has a ground-up loss of $150,000, applying the 100,000 xs 80,000 layer, results in a reinsurer's loss of 70,000.

Perform similar calculations for all policies over the last five years. Here is a table illustrating these calculations (current year is assumed to be *t*.)

Previous Years	Incurred Loss (fgu)	XL Reinsurer's Loss (relative to selected layer, 100,000 xs 80,000.
t-1 (last year)		
Policy 1	60,000	0
Policy 2	132,000	52,000
Policy 3	200,134	100,000
t-2 (two years ago)		
Policy 1	110,156	30,156
Policy 2	180,000	100,000
t-3 (three years ago)		
Policy 1	170,278	90,278
t-4 (four years ago)		
Policy 1	90,190	10,190
t-5 (5 years ago)		
Policy 1	80,586	586
Policy 2	185,674	100,000
Policy 3	158,133	78,133
		Sum = 561,343

Table 5.1

Second Step

The second step in the process is to obtain the total premium corresponding to all policies in the above table. For this example, the total premium is 7, 818, 148. This permits the calculation of the XL reinsurer's loss ratio applicable to the layer 100,000 xs 80,000.

The XL loss ratio = XL reinsurer's loss ratio / Total premium = 561,343 / 7,850,951 = 7.15%. This is called the *pure* burn cost ratio.

The pure burn cost ratio is called a *technical* reinsurance rate.

However, there are important shortcomings of the burn cost method that include:

a) In practice, the historical data on ground-up losses and associated premiums is modified to reflect predicted inflation and trends in the data. The objective in this section is to describe and illustrate the burn cost method. The suitability of historical data should always be assessed and appropriately adjusted to reflect the future reinsurance term. The theory of credibility is useful in this context and was briefly discussed in the third eBook entitled *Non-Life Insurance.* A complete treatment of the theory of credibility will be covered in Level II of the Art of Insurance series.
b) In addition, in Table 5.1 we calculated the XL reinsurer's losses from the layer 100,000 xs 80,000 without any consideration of serving to its benefit (inuring). But as pointed out by Clark (2014), *for experience rating, the only accurate way to reflect this underlying reinsurance is to restate the historical loss experience on a basis net of inuring reinsurance.*

Third Step 3

The final step in experience rating is to add a load to the technical reinsurance rate obtained in step 2. The z load is intended to cover the reinsurer's expenses and target profit margin. It is customary to denote this load a multiplier of the form *100/80 or 100/85 etc.* Hence, if the technical reinsurance rate (in step 2) is 7.15% and a load of 100/80 is applied, the *final rate* is 7.15% × 100/80 = 7.15% × 1.25 = 8.9375%. This is the rate that will be applied to equation (5.2).

Example 4

Objective: Obtain the XL reinsurance premium for a specified layer for a one-year term.

Data Provided on the reinsurance policy effective date based on the Experience Rating Method:

Subject premium = 5,342,190;
Adjustments for lapse and cancelled policies and serving to the benefit of the reinsurer = 30,135;
Technical reinsurance rate = 4.3%;
Load for reinsurer's expenses and target profit = 100/85;

Answer

Adjusted Subject Premium = 5,342,190 – 30,135 = 5,312,055.
Final Reinsurance Rate = Technical Rate × 100/85 = 4.3 % x 1.176471 = 5.0588%.

Using (5.1) XL Reinsurance Premium = 5,312,055 × .050588 = 268,727.5

This concludes this eBook.

REFERENCES

1. Albrecher, Hansjorg e*t al: Reinsurance Pricing,* Reinsurance: Actuarial and Statistical Aspects, Wiley Series in Probability and Statistics, 2017.
2. Bernard, Carole: *Risk Sharing and Pricing in the Reinsurance Market,* Handbook of Insurance, Chapter 21, 2013.
3. Best's Special Report on A.M. Best's Top 50 Ranking of Global Reinsurers in *Best's Insurance News and Analysis*, September 7, 2017.
4. Borch, Karl: *Equilibrium in a Reinsurance Market*, Econometrica, Vol.30, No.3, 1962.
5. Charpentier, Arthur: *Reinsurance, ruin and solvency issues: some pitfalls,* cahier de recherche 2010-06.
6. Chester, Ari *et al: Global Reinsurance: fit for the future?* McKinsey & Company, 2017.
7. Clark, David: *Basics of Reinsurance Pricing*, Actuarial Study Note, Revised 2014.
8. Eves, Michael *et al: Non-proportional Reinsurance,* International Actuarial Association Risk Book, Chapter 6, 2015.
9. Garven, James and Joan Lamm-Tennant: *The Demand for Reinsurance: Theory and Empirical Tests,* Insurance and Risk Management, Vol.7, No.3, 2003.
10. 1Jones, Gavin: *Longevity Risk and Reinsurance,* Reinsurance *News*, July 2013, Issue 76013.
11. Hart, David *et al: The Actuarial Practice of General Insurance,* Institute of Actuaries of Australia, Sydney, 1996.
12. Neuthor Laila: *Intermediation in Reinsurance Markets,* Verlag Versicherungswirtschaft, GmbH, Karlsruhe, 2013.
13. Plantin, Guillaume: *Does Reinsurance Need Reinsurers?* Journal of Risk and Insurance, Vol. 73, Issue 1, 2006.
14. Schwepcke, Andreas and Dieter Arndt: *Reinsurance, Principles and State of the Art–A guidebook for home learners,* Verlag Versicherungswirtschaft, 2004.