Testul 1

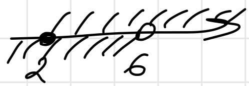
1) $<$; 2) $(0;+\infty)$ 3) $\frac{A_1}{A_2}=\frac{a^2}{\frac{\pi a^2}{4}}=\frac{4}{\pi}$

4) $\sqrt[3]{-8}^{\,-1}+\log_{27}3=-2^{-1}+\frac{1}{3}=-\frac{1}{2}+\frac{1}{3}=-\frac{1}{6}$ R/s: $-\frac{1}{6}$

5) $d=-6-8-10+10+4+12=2$

$\sqrt{x-2}<2$ $x-2<4$

$DVA=[2;+\infty)$ $x<6$



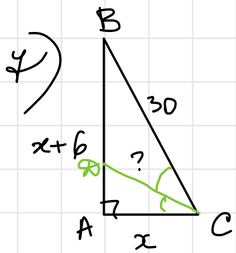
R/s: $S=[2;6)$

6) $(1+2i)^2=-(y+xi)i \Rightarrow 1+4i-4=-yi+x \Rightarrow 4i-3=-yi+x \Rightarrow$

$\Rightarrow \begin{cases} y=-4 \\ x=-3 \end{cases} \Rightarrow z=-3-4i$

R/s: $z=-3-4i$

7)



Th. Pitagora în $\triangle ABC$: $(x+6)^2+x^2=900 \Rightarrow 2x^2+12x+36=900$

$x^2+6x-432=0 \Rightarrow x_1=-24$, $x_2=18 \Rightarrow x=18 \Rightarrow AC=18\text{ cm}$, $AB=24\text{ cm}$

Proprietatea bisectoarei: $\frac{AD}{AC}=\frac{DB}{BC} \Rightarrow \frac{AD}{\cancel{18}_3}=\frac{24-AD}{\cancel{30}_5} \Rightarrow$

$\Rightarrow 5AD=72-3AD \Rightarrow AD=9\text{ cm}$

Th. Pitagora în $\triangle ADC$: $DC^2=9^2+18^2=5\cdot 9^2 \Rightarrow DC=9\sqrt{5}\text{ cm}$.

R/s: $9\sqrt{5}$ cm

8) $y=-x+1$; $f:\mathbb{R}\to\mathbb{R}$, $f(x)=x+e^{-2x}$

$f'(x)=1-2e^{-2x}$

$f'(x_0)=-1 \Rightarrow 1-2e^{-2x_0}=-1 \Rightarrow e^{-2x_0}=1 \Rightarrow x_0=0$

$f(x_0)=1 \Rightarrow$ punctul de tangență are coord. $(0;1)$

R/s: $A(0;1)$

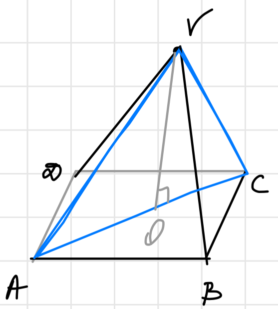
Testul 1

9) $\left.\begin{array}{l} 5\text{ b.a.} \\ 7\text{ b.n.} \end{array}\right\} \xrightarrow{3\text{ b.}}$ $A = \{aaa\}$

$$P(A) = \frac{C_5^3}{C_{12}^3} = \frac{5!}{3!\cdot 2!} \cdot \frac{9!\cdot 3!}{12!} = \frac{1}{22}$$

R/s: $\frac{1}{22}$

10)



VAC − Δ echilat. $\Rightarrow VO = AC\frac{\sqrt{3}}{2} \Rightarrow$

$\Rightarrow \frac{AC\sqrt{3}}{2} = 4\sqrt{3} \Rightarrow AC = 8\text{ cm} \Rightarrow$

$\Rightarrow AB\sqrt{2} = 8 \Rightarrow AB = 4\sqrt{2}\text{ cm}$

$$V_{piz} = \frac{1}{3}\cdot AB^2\cdot VO = \frac{1}{3}\cdot 32\cdot 4\sqrt{3} = \frac{128\sqrt{3}}{3}(\text{cm}^3)$$

R/s: $\frac{128\sqrt{3}}{3}\text{ cm}^3$

11) $F(x) = \int f(x)\,dx = -4\cos x + C$

$F\left(\frac{\pi}{3}\right) = 0 \Rightarrow -4\cos\frac{\pi}{3} + C = 0 \Rightarrow -2 + C = 0 \Rightarrow C = 2.$

R/s: $F: \mathbb{R} \to \mathbb{R}$, $F(x) = -4\cos x + 2$

12) $(\sqrt{x-3} - 2)(x-a) = 0$, $|S| = 1$

DVA $= [3; +\infty)$

$$\left[\begin{array}{l} \sqrt{x-3} - 2 = 0 \\ x - a = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = 7 \in \text{DVA} \\ x = a \end{array}\right.$$

$|S| = 1 \Rightarrow a \notin \text{DVA} \Rightarrow a < 3$

R/s: $a \in (-\infty; 3)$

Testul 2

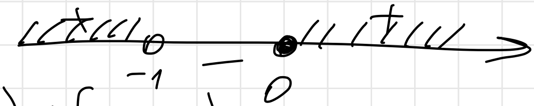
1) $\frac{1}{e^2} < \frac{1}{2\sqrt{e}}$; 2) pară ; 3) 40°

4) $\sqrt[3]{3+5} = \sqrt[3]{8} = 2$

R/s: 2

5) $\frac{3-x}{x+1} \le 3 \Rightarrow \frac{3-x-3x-3}{x+1} \le 0 \Rightarrow \frac{-4x}{x+1} \le 0 \Rightarrow$

DVA $= \mathbb{R} \setminus \{-1\}$ $\Rightarrow \frac{4x}{x+1} \ge 0$

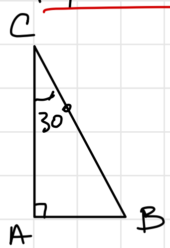


R/s: $S = (-\infty; -1) \cup [0; +\infty)$

6) $d = 4 - 1 - 2 - 2 = -1$; $(z+1)^2 + 1 = 0 \Rightarrow \left[\begin{array}{l} z+1 = i \\ z+1 = -i \end{array}\right. \Rightarrow \left[\begin{array}{l} z = -1+i \\ z = -1-i \end{array}\right.$

R/s: $S = \{-1+i; -1-i\}$

7)



$R = 12\text{ cm} \Rightarrow BC = 24\text{ cm}$

$m(\angle C) = 30^\circ \Rightarrow AB = \frac{1}{2} BC = 12\text{ cm}$

$AC = AB\sqrt{3} = 12\sqrt{3}\text{ cm}$

$\mathcal{A}_{\triangle ABC} = \frac{AB \cdot AC}{2} = \frac{12 \cdot 12\sqrt{3}}{2} = 72\sqrt{3}\ (\text{cm}^2)$

R/s: $72\sqrt{3}\text{ cm}^2$

8) $F(x) = x + \frac{x^2}{2} - \frac{1}{2}\cos 2x + C$

$F(0) = 1 \Rightarrow -\frac{1}{2} + C = 1 \Rightarrow C = \frac{3}{2}$.

R/s: $F: \mathbb{R} \to \mathbb{R}$, $F(x) = x + \frac{x^2}{2} - \frac{1}{2}\cos 2x + \frac{3}{2}$

Testul 2

9) $A_1 = \{$primul prieten este angajat$\}$

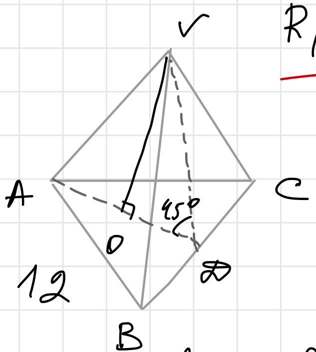
$A_2 = \{$al doilea prieten este angajat$\}$

$A = \{$cel puțin un prieten este angajat$\}$.

$$P(A) = 1 - P(\overline{A_1}) \cdot P(\overline{A_2}) = 1 - \frac{1}{4} \cdot \frac{1}{4} = 1 - \frac{1}{16} = \frac{15}{16}$$

R/S: $\frac{15}{16}$

10)



$$AD = \frac{AB\sqrt{3}}{2} = 6\sqrt{3}\text{ cm}.$$

$$OD = \frac{1}{3} AD = 2\sqrt{3}\text{ cm}$$

ΔVOD - isoscel $\Rightarrow VO = OD = 2\sqrt{3}$ cm.

$VD = OD\sqrt{2} = 2\sqrt{6}$ cm.

$$A_{tot} = A_b + A_{lat} = \frac{AB^2\sqrt{3}}{4} + \frac{VD \cdot 3AB}{2} =$$

$$= \frac{144\sqrt{3}}{4} + \frac{2\sqrt{6} \cdot 36}{2} = 36\sqrt{3} + 36\sqrt{6} = 36(\sqrt{3}+\sqrt{6})(cm^2)$$

$$V_{pir} = \frac{1}{3} \cdot A_b \cdot VO = \frac{1}{3} \cdot 36\sqrt{3} \cdot 2\sqrt{3} = 24\sqrt{6}(cm^3)$$

R/S: $36(\sqrt{3}+\sqrt{6})$ cm^2; $24\sqrt{6}$ cm^3.

Testul 2

11) $\sqrt{1-\cos^2 x} - (\cos^2 x)' = \lg \operatorname{tg} \frac{5\pi}{4}$

$|\sin x| + 2\cos x \cdot \sin x = 0$

a) $\sin x \geq 0 \Rightarrow \sin x(1+2\cos x)=0 \Rightarrow \left[\begin{array}{l}\sin x = 0\\ \cos x = -\frac{1}{2}\end{array}\right. \Rightarrow$

$\Downarrow$

$x \in [2k\pi; \pi+2k\pi]$
$k \in \mathbb{Z}$

$\xRightarrow{\sin x \geq 0} \left[\begin{array}{l}x = m\pi,\ m\in\mathbb{Z}\\ x = \frac{2\pi}{3} + 2n\pi,\ n\in\mathbb{Z}\end{array}\right.$

b) $\sin x < 0 \Rightarrow \sin x(2\cos x - 1) = 0 \Rightarrow \left[\begin{array}{l}\sin x = 0\\ \cos x = \frac{1}{2}\end{array}\right. \Rightarrow$

$x \in (\pi+2k\pi; 2\pi+2k\pi)$
$k\in\mathbb{Z}$

$\xRightarrow{\sin x < 0} x = -\frac{\pi}{3} + 2l\pi,\ l\in\mathbb{Z}$

R/S: $S = \left\{m\pi; \frac{2\pi}{3} + 2n\pi; -\frac{\pi}{3} + 2l\pi \mid m,n,l\in\mathbb{Z}\right\}$

12) $2\log_3^2 x - |\log_3 x| + a = 0$ $\quad |S| = 4.$

DVA $= \mathbb{R}_+^*$

Notăm: $|\log_3 x| = t,\ t>0$

$2t^2 - t + a = 0$

$\Delta = 1 - 8a$

$\Delta > 0 \Rightarrow a < \frac{1}{8}$ $(*)$

$t_1 = \frac{1-\sqrt{1-8a}}{4} > 0$

$t_2 = \frac{1+\sqrt{1-8a}}{4} > 0,\ \forall a\in(-\infty; \frac{1}{8})$

$t_1 > 0 \Rightarrow \sqrt{1-8a} < 1 \Rightarrow$
$\Rightarrow 1-8a<1 \Rightarrow a>0$ $(**)$

Din $(*)$ și $(**) \Rightarrow a\in(0;\frac{1}{8})$

R/S: $a\in(0;\frac{1}{8})$

$(*)$ $a\in(-\infty;0) \Rightarrow |S| = 2$

$a = \frac{1}{8} \Rightarrow |S| = 2$

$a > \frac{1}{8} \Rightarrow S = \emptyset$

$a = 0 \Rightarrow |S| = 3$

Testul 3

1) $\frac{5}{4}$; 2) -5; 3) 120°; 4) $\frac{100}{25} = 4$; R/S: 4

5) $4^{-3x-6} \le 2^{-x} \cdot 8$

$2^{-6x-12} \le 2^{-x+3} \Rightarrow -5x \le 15 \Rightarrow x \ge -3$

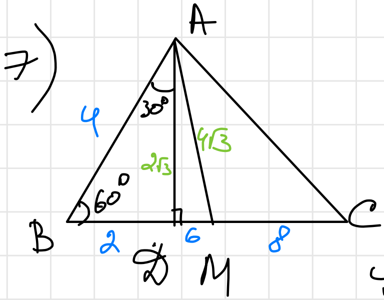
R/S: $S = [-3; +\infty)$

6) $z = -pi + (2+i)^2 - p = -pi + 4 + 4i - 1 - p = 3 - p + (4-p)i$

z - pur imag. $\Rightarrow \operatorname{Re} z = 0 \Rightarrow 3 - p = 0 \Rightarrow p = 3.$

R/S: $p = 3$

7)



$AM = 4\sqrt{3}$ cm; $AD = 2\sqrt{3}$ cm

$m(\angle BAD) = 30^\circ \Rightarrow BD = \frac{AD}{\sqrt{3}} = 2$ cm

$AB = 2BD = 4$ cm

Th. Pitagora în ΔAMD: $MD^2 = 48 - 12 = 36$

$MD = 6$ cm

AM - mediană $\Rightarrow MC = MB = MD + DB = 8$ cm.

Th. Pitagora în ΔADC: $AC^2 = 12 + 196 = 208 \Rightarrow AC = 4\sqrt{13}$ cm

$P_{\Delta ABC} = (20 + 4\sqrt{13})$ cm; $\mathcal{A}_{\Delta ABC} = \frac{AD \cdot BC}{2} = \frac{2\sqrt{3} \cdot 16}{2} = 16\sqrt{3}\,(\text{cm}^2)$

R/S: $(20 + 4\sqrt{13})$ cm; $16\sqrt{3}$ cm^2

8) $f'(x) = \ln x + \frac{x+1}{x} - 2$; $f''(x) = \frac{1}{x} + \frac{x - x - 1}{x^2} = \frac{1}{x} - \frac{1}{x^2}$

$\frac{1}{x} - \frac{1}{x^2} = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1 \in D(f)$

R/S: $S = \{1\}$.

Testul 3

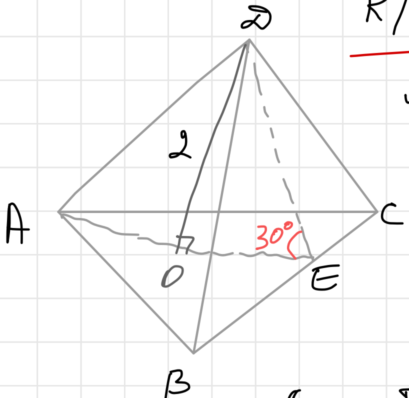
9) $\left.\begin{matrix} 4f. \\ 6b. \end{matrix}\right\} \xrightarrow{4\text{ elevi}} \quad A = \{4f\} \cup \{1b; 3f\} \cup \{2b; 2f\}$

$$P(A) = \frac{C_4^4 + C_6^1 \cdot C_4^3 + C_6^2 \cdot C_4^2}{C_{10}^4} = \frac{1 + 6\cdot 4 + \frac{6!}{4!\cdot 2!} \cdot \frac{4!}{2!\cdot 2!}}{\frac{10!}{6!\cdot 4!}} =$$

$$= \frac{25 + 90}{210} = \frac{115}{210} = \frac{23}{42}$$

R/s: $\frac{23}{42}$

10)



$m(\angle DEO) = 30^\circ \Rightarrow OE = DO\sqrt{3} = 2\sqrt{3}\,cm$

$DE = 2DO = 4\,cm$

ΔABC - echilateral $\Rightarrow AE = 3OE = 6\sqrt{3}\,cm$

$AE = \frac{AB\sqrt{3}}{2} \Rightarrow AB = \frac{2AE}{\sqrt{3}} = \frac{12\sqrt{3}}{\sqrt{3}}$

$AB = 12\,cm.$

$$A_{lat} = \frac{P_{ABC} \cdot DE}{2} = \frac{36 \cdot 4}{2} = 72\,(cm^2)$$

R/s: $72\,cm^2$

11) $3\sin x = 2\cos^2 x \Rightarrow 3\sin x = 2 - 2\sin^2 x \Rightarrow$

$\Rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

Notăm: $\sin x = t;\ t \in [-1; 1]$

$2t^2 + 3t - 2 = 0$

$t_1 = 2 > 1$

$t_2 = \frac{1}{2} \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = (-1)^k \cdot \frac{\pi}{6} + k\pi,\ k \in \mathbb{Z}$

$$\begin{cases} x = (-1)^k \cdot \frac{\pi}{6} + k\pi,\ k \in \mathbb{Z} \\ x \in \left(\frac{\pi}{2}; \pi\right) \end{cases} \Rightarrow x = \frac{5\pi}{6}$$

R/S: $S = \left\{\frac{5\pi}{6}\right\}$

Testul 3

12) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = ax - \ln(x^2+1)$

$$f'(x) = a - \frac{2x}{x^2+1} = \frac{ax^2-2x+a}{x^2+1}$$

$f \searrow$ pe $\mathbb{R} \Rightarrow f'(x) < 0$, $\forall x \in \mathbb{R}$.

$\frac{ax^2-2x+a}{x^2+1} < 0$, $\forall x \in \mathbb{R} \Rightarrow ax^2-2x+a < 0$, $\forall x \in \mathbb{R}$

a) $a = 0 \Rightarrow -2x < 0$, $\forall x \in \mathbb{R} \to$ F

b) $a > 0$: $ax^2 - 2x + a < 0$, $\forall x \in \mathbb{R} \Rightarrow a \in \emptyset$

c) $a < 0$: $ax^2 - 2x + a < 0$, $\forall x \in \mathbb{R} \Rightarrow \Delta < 0$

$\Delta = 4 - 4a^2 = 4(1-a^2)$

$\Delta < 0 \Rightarrow 1 - a^2 < 0 \Rightarrow a \in (-\infty; -1) \cup (1; +\infty)$

$$\begin{cases} a < 0 \\ a \in (-\infty; -1) \cup (1; +\infty) \end{cases} \Rightarrow a \in (-\infty; -1)$$

R/s: $a \in (-\infty; -1)$

Testul 4

1) $\frac{1}{3}$; 2) $f(3) > f(\pi)$; 3) 80°

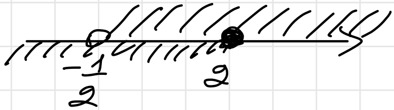
4) $\sqrt{64^{\frac{2}{3}} + \left(\frac{1}{3}\right)^{-2}} = \sqrt{4^{3\cdot\frac{2}{3}} + \left(\frac{3}{1}\right)^{2}} = \sqrt{16+9} = \sqrt{25} = 5$

R/S: 5

5) $\mathcal{D}(x) = \frac{x-2}{\sqrt{2x+1}} + 1$; $\mathcal{D}(x) \le 1 \Rightarrow \frac{x-2}{\sqrt{2x+1}} + 1 \le 1$

$\frac{x-2}{\sqrt{2x+1} > 0} \le 0 \Rightarrow x - 2 \le 0 \Rightarrow \underline{x \le 2}$

DVA: $\left(-\frac{1}{2}; +\infty\right)$



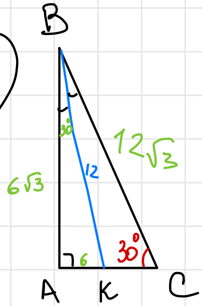
R/S: $S = \left(-\frac{1}{2}; 2\right]$

6) $w = pi - (1+i)^2 + p = pi - 1 - 2i + 1 + p = p + (p-2)i$

$w \in \mathbb{R} \Rightarrow \operatorname{Im} z = 0 \Rightarrow p - 2 = 0 \Rightarrow p = 2$

R/S: $p = 2$

7)



BK – bisectoare; $m(\angle ABC) = 60^\circ$ $\Rightarrow m(\angle ABK) = \frac{1}{2} m(\angle ABC) = 30^\circ \Rightarrow$

$\Rightarrow AK = \frac{1}{2} BK = 6$ cm

$AB = AK\sqrt{3} = 6\sqrt{3}$ cm.

$m(\angle ACB) = 30^\circ \Rightarrow BC = 2AB = 12\sqrt{3}$ cm.

BK – bisectoare $\Rightarrow \frac{KA}{AB} = \frac{KC}{BC} \Rightarrow \frac{6}{\cancel{6\sqrt{3}}_{1}} = \frac{KC}{\cancel{12\sqrt{3}}_{2}} \Rightarrow KC = 12$ cm

$\mathcal{A}_{\Delta ABC} = \frac{AB \cdot AC}{2} = \frac{6\sqrt{3} \cdot 18}{2} = 54\sqrt{3}\ (\text{cm}^2)$

R/S: $54\sqrt{3}$ cm^2

Testul 4

8) $f'(x) = 2\cos 2x - 2$; $f'(x) = 0 \Rightarrow 2\cos 2x - 2 = 0 \Rightarrow \cos 2x = 1 \Rightarrow$
$\Rightarrow 2x = k\pi,\ k \in \mathbb{Z} \Rightarrow x = \frac{k\pi}{2},\ k \in \mathbb{Z}$

x	$-\frac{\pi}{2}$		$\frac{\pi}{2}$
f'		$-$	
f	max	$\longrightarrow$	min

$f_{max}\left(-\frac{\pi}{2}\right) = \pi$

$f_{min}\left(\frac{\pi}{2}\right) = -\pi$

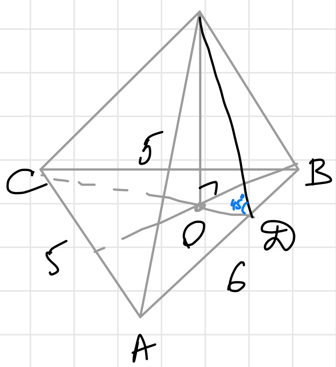
R/S: $f_{max}\left(-\frac{\pi}{2}\right) = \pi$; $f_{min}\left(\frac{\pi}{2}\right) = -\pi$

9) $A = \{$se extrag fișele A, L, I, N, A$\}$

$P(A) = \frac{\cancel{2}^{1}}{7} \cdot \frac{2^{1}}{\cancel{6}_{3}} \cdot \frac{2}{5} \cdot \frac{1}{\cancel{4}_{1}} \cdot \frac{1}{3} = \frac{1}{315}$

R/S: $\frac{1}{315}$

10)



$AC = BC = 5cm$; $AB = 6cm$

Unghiurile diedre de la bază sunt congruente $\Rightarrow O$ este centrul cercului înscris în $\Delta ABC \Rightarrow OD = r$

ΔODV - isoscel $\Rightarrow VO = OD = r$.

$r = \frac{\mathcal{A}_{\Delta ABC}}{p_{\Delta ABC}} = \frac{12}{8} = \frac{3}{2}\ (cm) \Rightarrow VO = \frac{3}{2}\ cm$

$p_{\Delta ABC} = \frac{5+5+6}{2} = 8\ (cm)$; $\mathcal{A}_{\Delta ABC} = \sqrt{8 \cdot 3 \cdot 3 \cdot 2} = 12\ (cm^2)$

$V_{piz} = \frac{1}{3} \cdot \mathcal{A}_{\Delta ABC} \cdot VO = \frac{1}{\cancel{3}_{1}} \cdot \cancel{12}^{6} \cdot \frac{\cancel{3}^{1}}{\cancel{2}_{1}} = 6\ (cm^3)$

R/S: $6\ cm^3$.

Testul 4

11) $$\int_1^3 (|x+1| + |x-2|)\,dx = \int_1^3 |x+1|\,dx + \int_1^3 |x-2|\,dx =$$

$$= \int_1^3 (x+1)\,dx - \int_1^2 (x-2)\,dx + \int_2^3 (x-2)\,dx =$$

$$= \left(\frac{x^2}{2} + x\right)\Big|_1^3 - \left(\frac{x^2}{2} - 2x\right)\Big|_1^2 + \left(\frac{x^2}{2} - 2x\right)\Big|_2^3 =$$

$$= \frac{9}{2} + 3 - \cancel{\frac{1}{2}} - 1 - \cancel{2} + 4 + \cancel{\frac{1}{2}} - 2 + \frac{9}{2} - 6 - \cancel{2} + \cancel{4} = 9 - 2 = 7$$

R/S: 7

12) $5 \cdot 3^{x+1} - m = 10(2 - m \cdot 3^x)$, $S = \emptyset$

$15 \cdot 3^x - m = 20 - 10m \cdot 3^x$

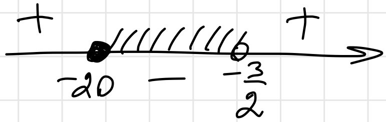
$15 \cdot 3^x + 10m \cdot 3^x = 20 + m$

$(15 + 10m) \cdot 3^x = 20 + m$

a) $15 + 10m = 0 \Rightarrow m = -\frac{3}{2} \Rightarrow 0 \cdot 3^x = 20 - \frac{3}{2} \Rightarrow F \Rightarrow S = \emptyset$

b) $15 + 10m \neq 0 \Rightarrow 3^x = \dfrac{20+m}{15+10m} \overset{S=\emptyset}{\Longrightarrow} \dfrac{20+m}{15+10m} \leq 0$

$\dfrac{20+m}{15+10m} = 0 \Rightarrow m = -20$



$m \in \left[-20; -\frac{3}{2}\right)$

R/S: $m \in \left[-20; -\frac{3}{2}\right]$.

Testul 5

1) $2^{\log_4 9} = 2^{\frac{1}{2}\cdot 2\log_2 3} = 3 = \sqrt[3]{27}$; 2) $E(f) = (-\infty; 2]$

3) $m(\angle AKC) = 180^\circ - 11x^\circ$

ΔACK: $180^\circ - 11x^\circ + 20^\circ = 90^\circ \Rightarrow 11x^\circ = 110^\circ \Rightarrow x = 10$

4) $0{,}5\log_3 25 - 2\log_3\sqrt{5} = \log_3 25^{\frac{1}{2}} - \log_3(\sqrt{5})^2 =$

$= \log_3 5 - \log_3 5 = 0$ R/s: 0

5) $\sqrt{2-x} < 1$; DVA $= (-\infty; 2]$

$2 - x < 1$

$x > 1$

$1 \qquad 2$

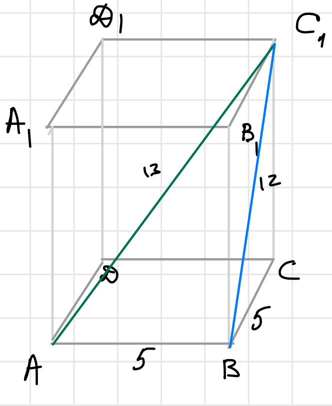
R/s: $S = (1; 2]$

6) $d = -2 + 2 - 2 - 2 = -4$

$(z-i)^2 - 4 = 0 \Rightarrow (z-i)^2 = 4 \Rightarrow \left[\begin{array}{l} z - i = 2 \\ z - i = -2 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} z = 2 + i \\ z = -2 + i \end{array}\right.$

R/s: $S = \{2 + i; -2 + i\}$

7)



Th. lui Pitagora în ΔABC_1:

$AB^2 = 169 - 144 = 25 \Rightarrow AB = 5$ cm

Th. lui Pitagora în ΔBC_1C:

$C_1C^2 = 144 - 25 = 119 \Rightarrow C_1C = \sqrt{119}$ cm

$A_{lat} = P_b \cdot C_1C = 4 \cdot 5 \cdot \sqrt{119} = 20\sqrt{119}\,(cm^2)$

R/s: $20\sqrt{119}$ cm^2

Testul 5

8) $F(x)=2x^3-\ln|x|+x+C$; $A(-1;2)\in G_F \Rightarrow F(-1)=2 \Rightarrow$

$\Rightarrow -2-\ln 1-1+C=2 \Rightarrow C=5$

R/s: $F:(-\infty;0)\to\mathbb{R}$, $F(x)=2x^3-\ln|x|+x+5$

9) $A=\{$fața cu 2 pct. va apărea exact de 2 ori$\}$

$P(A)=C_4^2\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}$

$\boxed{2p}\ \boxed{2p}\ \boxed{}\ \boxed{}$

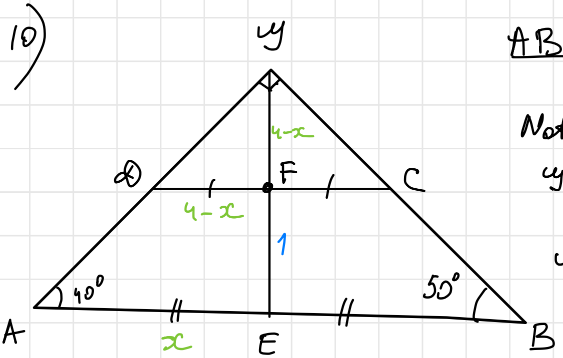
1,3-6p 1,3-6p.

$P(A)=\frac{\cancel{4!}^{\,2\cdot 3}}{\cancel{2!}_{\,1}\cdot\cancel{2!}_{\,1}}\cdot\frac{1}{36}\cdot\frac{25}{36}$

$P(A)=\frac{25}{216}$

R/s: $\frac{25}{216}$

10)



$\frac{AB+CD}{2}=4 \Rightarrow \frac{AB}{2}+\frac{CD}{2}=4$

Notăm: $\frac{AB}{2}=AE=x \Rightarrow DF=\frac{CD}{2}=4-x$

GF - mediană în ΔDGC - dreptung. $\Rightarrow$

$\Rightarrow GF=DF=4-x$

GF - mediană în ΔABG - dreptunghic $\Rightarrow$

$\Rightarrow GE=AE \Rightarrow 4-x+1=x \Rightarrow x=\frac{5}{2}$

$AB=2x=5$ cm

$CD=2(4-x)=8-2x=8-5=(3\text{ cm})$

R/s: 5 cm; 3 cm

Testul 5

11) $f'(x) = -2\sin x + \sqrt{3}$; $f'(x) = 0 \Rightarrow -2\sin x + \sqrt{3} = 0 \Rightarrow$

$\Rightarrow \sin x = \dfrac{\sqrt{3}}{2} \xrightarrow{D(f)=[0;\frac{\pi}{2}]} x = \dfrac{\pi}{3}$

x	0		$\frac{\pi}{3}$		$\frac{\pi}{2}$
f'		$+$		$-$	
f		$\nearrow$	1 max	$\searrow$	

$f\left(\frac{\pi}{3}\right) = 2\cdot\frac{1}{2} + \frac{\sqrt{3}\pi}{3} - \frac{\sqrt{3}\pi}{3} = 1$

R/s: $f_{max}\left(\frac{\pi}{3}\right) = 1$

12) $x^2 - (3a-1)\cdot|x| + 2a^2 - a = 0$, $\quad |S| = 3$

a) $2a^2 - a = 0 \Rightarrow a(2a-1) = 0 \Rightarrow \left[\begin{array}{l} a = 0 \\ a = \frac{1}{2} \end{array}\right.$

$a = 0 \Rightarrow x^2 + |x| = 0 \Rightarrow |x|\cdot(|x|+1) = 0 \Rightarrow \left[\begin{array}{l} |x| = 0 \\ |x| = -1 \end{array}\right. \Rightarrow x = 0.$
$|S| = 1$

$a = \frac{1}{2} \Rightarrow x^2 - \frac{1}{2}|x| = 0 \Rightarrow |x|\cdot\left(|x| - \frac{1}{2}\right) = 0 \Rightarrow \left[\begin{array}{l} |x| = 0 \\ |x| = \frac{1}{2} \end{array}\right. \Rightarrow \left[\begin{array}{l} x = 0 \\ x = \pm\frac{1}{2} \end{array}\right.$
$|S| = 3$

b) $2a^2 - a \neq 0$: $|x|^2 - (3a-1)|x| + 2a^2 - a = 0$

$\Delta = (3a-1)^2 - 4(2a^2 - a) = 9a^2 - 6a + 1 - 8a^2 + 4a = a^2 - 2a + 1$

$\Delta = (a-1)^2 \geq 0$, $\forall a \in \mathbb{R}$.

$a = 1 \Rightarrow |x|^2 - 2|x| + 1 = 0 \Rightarrow (|x| - 1)^2 = 0 \Rightarrow |x| = 1 \Rightarrow x = \pm 1$
$|S| = 2$

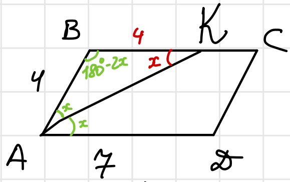
$a \in \mathbb{R} \setminus \{1; \frac{1}{2}; 0\} \Rightarrow \left[\begin{array}{l} |x| = \frac{3a-1+a-1}{2} \\ |x| = \frac{3a-1-a+1}{2} \end{array}\right. \Rightarrow \left[\begin{array}{l} |x| = \frac{4a-2}{2} \\ |x| = \frac{2a}{2} \end{array}\right. \Rightarrow \left[\begin{array}{l} |x| = 2a-1 \\ |x| = a \end{array}\right.$

Fiindă $a \notin \{0; \frac{1}{2}; 1\}$ ultima totalitate are 2, 4 sau nici o soluție

R/s: $a = \frac{1}{2}$

Testul 6

1) $2^{-3} = \sqrt{\frac{1}{64}}$; 2) $f \searrow$ pe $[-1; 3]$; 3)



$KC = 3$ cm

4) $\sqrt[3]{25^{\log_5 4} + 4\log_{\frac{1}{3}} 9} = \sqrt[3]{16 - 4\log_3 9} = \sqrt[3]{16-8} = \sqrt[3]{8} = 2$

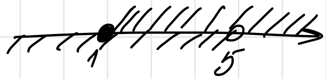
R/S: 2

5) $D(x) = 2\sqrt{x-1} - \sqrt{x-1} = \sqrt{x-1}$

$\sqrt{x-1} < 2$

$DVA = [1; +\infty)$

$x - 1 < 4 \Rightarrow x < 5$



R/S: $S = [1; 5)$

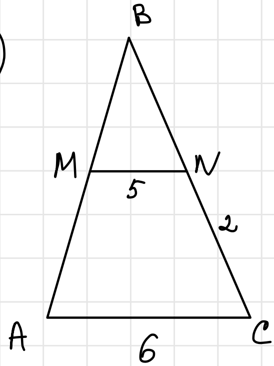
6) $(1+i)(x-i) = -(1-i)(y+3i) \Rightarrow x - i + ix + 1 = -y - 3i + yi - 3$

$\Rightarrow \begin{cases} x+1 = -y-3 \\ x-1 = y-3 \end{cases} \Leftrightarrow \begin{cases} x+y = -4 \\ x-y = -2 \end{cases} \Leftrightarrow \begin{cases} 2x = -6 \\ y = -4-x \end{cases} \Leftrightarrow \begin{cases} x = -3 \\ y = -1 \end{cases}$

$x + y = -3 + (-1) = -4$

R/S: -4

7)



$MN \parallel AC \Rightarrow \frac{BN}{BC} = \frac{MN}{AC} \Rightarrow \frac{BN}{2+BN} = \frac{5}{6} \Rightarrow$

$\Rightarrow 6BN = 10 + 5BN \Rightarrow BN = 10$ cm $\Rightarrow$

$\Rightarrow BC = 12$ cm

ΔABC - isoscel $\Rightarrow AB = BC = 12$ cm.

$p_{\Delta ABC} = \frac{12+12+6}{2} = 15$ (cm)

$\mathcal{A}_{\Delta ABC} = \sqrt{15 \cdot 3 \cdot 3 \cdot 9} = 9\sqrt{15}$ (cm^2)

R/S: $9\sqrt{15}$ cm^2

Testul 6

8) $f'(x)=\frac{7}{2\sqrt{7x-6}}$; $d: 12y-7x-4=0$

$y=\frac{7}{12}x+\frac{1}{3}$

Tangenta este paralelă cu $d \Rightarrow f'(x_0)=\frac{7}{12} \Rightarrow \frac{7}{2\sqrt{7x_0-6}}=\frac{7}{12} \Rightarrow$

$\Rightarrow \sqrt{7x_0-6}=6 \Rightarrow 7x_0-6=36 \Rightarrow 7x_0=42 \Rightarrow x_0=6$

$f(6)=\sqrt{42-6}=6$; $y=f'(x_0)(x-x_0)+f(x_0) \Rightarrow$

$\Rightarrow y=\frac{7}{12}(x-6)+6 \Rightarrow y=\frac{7}{12}x-\frac{7}{2}+6 \Rightarrow y=\frac{7}{12}x+\frac{5}{2}$

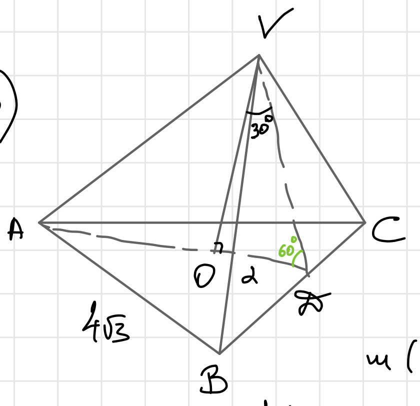
R/s: $y=\frac{7}{12}x+\frac{5}{2}$

9) $A=\{$din 3 elevi exact 2 nu vor susține examenul$\}$

$P(A)=3\cdot 0{,}1\cdot 0{,}1\cdot 0{,}9=0{,}027$

R/s: $0{,}027$

10)



$R=4$ cm ; $R=AO=\frac{2}{3}\cdot AO=\frac{\cancel{2}}{\cancel{3}}\cdot\frac{AB\cancel{\sqrt{3}}}{\cancel{2}} \Rightarrow$

$\Rightarrow 4=\frac{AB}{\sqrt{3}} \Rightarrow AB=4\sqrt{3}$ cm

$OD=\frac{1}{2}\cdot AO=2$ cm

$m(\angle OVD)=30^\circ \Rightarrow VO=OD\sqrt{3}=2\sqrt{3}$ cm

$V_{piz}=\frac{1}{3}\cdot A_b\cdot VO=\frac{1}{\cancel{3}}\cdot\frac{\cancel{16}\cdot\cancel{3}}{\cancel{4}}\cdot 2\sqrt{3}=8\sqrt{3}\,(cm^3)$

R/s: $8\sqrt{3}$ cm^3

Testul 6

11) $\mathcal{I} = \int_1^2 \left((x+1)\ln x - 2(x-1) \right) dx = \int_1^2 (x+1)\ln x \, dx - (x-1)^2 \Big|_1^2 =$

$= \left(\left(\frac{x^2}{2} + x \right) \ln x - \frac{x^2}{4} - x - (x-1)^2 \right) \Big|_1^2 = 4\ln 2 - 1 - 2 - 1 + \frac{1}{4} + 1 = 4\ln 2 - \frac{11}{4}$

$\int (x+1)\ln x \, dx = \left| \begin{matrix} u = \ln x; \; du = \frac{1}{x} dx \\ dv = (x+1)dx; \; v = \frac{x^2}{2} + x \end{matrix} \right| = \left(\frac{x^2}{2} + x \right) \ln x - \int \left(\frac{x}{2} + 1 \right) dx =$

$= \left(\frac{x^2}{2} + x \right) \ln x - \frac{x^2}{4} - x + C$

R/S: $4\ln 2 - \frac{11}{4}$

12) $(x-a)\log_5(2x-1) = 0$, $|S| = 1$

$DVA = \left(\frac{1}{2}; +\infty \right)$

$\left[\begin{matrix} x - a = 0 \\ \log_5(2x-1) = 0 \end{matrix} \right. \Leftrightarrow \left[\begin{matrix} x = a \\ 2x - 1 = 1 \end{matrix} \right. \Leftrightarrow \left[\begin{matrix} x = a \\ x = 1 \in DVA \end{matrix} \right. \xRightarrow{|S|=1} a \notin DVA \Rightarrow$

$\Rightarrow a \le \frac{1}{2} \Rightarrow a \in \left(-\infty; \frac{1}{2} \right]$

R/S: $a \in \left(-\infty; \frac{1}{2} \right]$

Testul 7

1) -2; 2) 3; 3) 3; 4) $\log_2 0{,}064 + 3\log_2 10 = \log_2 0{,}064 + \log_2 1000 =$
$= \log_2 64 = 6$

R/s: 6

5) $5\cdot 5^{2x-2} - 1 < 4$

$5\cdot 5^{2x-2} < 5$

$5^{2x-2} < 1$

$2x - 2 < 0$

$x < 1$

R/s: $S = (-\infty; 1)$

6) $z = \overline{w} \Rightarrow (3-2i)x + (1-3i)y = 4+9i \Rightarrow$

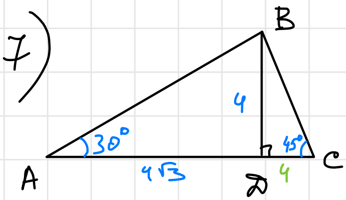
$\Rightarrow 3x - 2xi + y - 3yi = 4+9i$

$$\begin{cases} 3x + y = 4 \mid \cdot 3 \\ -2x - 3y = 9 \end{cases} \Leftrightarrow \begin{cases} 9x + 3y = 12 \\ -2x - 3y = 9 \end{cases} (+) \Leftrightarrow \begin{cases} 7x = 21 \\ 3x + y = 4 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x = 3 \\ 9 + y = 4 \end{cases} \Leftrightarrow \begin{cases} x = 3 \\ y = -5 \end{cases}$$

R/s: $x = 3; y = -5$

7)



ΔBDC – isoscel $\Rightarrow BD = DC = 4$ cm

$m(\angle A) = 30° \Rightarrow AD = BD\sqrt{3} = 4\sqrt{3}$ cm

$$\mathcal{A}_{\Delta ABD} = \frac{AD \cdot BD}{2} = \frac{4\sqrt{3}\cdot 4}{2} = 8\sqrt{3}\ (cm^2)$$

R/s: $8\sqrt{3}\ cm^2$

8) $f'(x) = 3x^2 - x - 10$

$3x^2 - x - 10 = 0$

$\Delta = 1 + 120 = 121$

$x_1 = \frac{1-11}{6} = -\frac{5}{3}$

$x_2 = 2$

x	$-\infty$		$-\frac{5}{3}$		2		$+\infty$
f'		$+$		$-$		$+$	
f		$\nearrow$	max	$\searrow$	min	$\nearrow$	

$f_{min}(2) = 8 - 2 - 20 + 2020 = 2006$

R/s: 2006

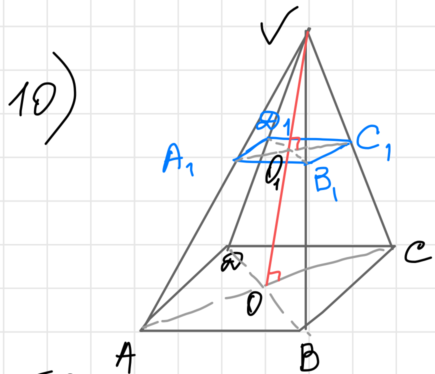
Testul 7

9) $A = \{$cel puțin una din companii acceptă cererea$\}$

$$P(A) = 1 - P(\overline{A}) = 1 - 0{,}25 \cdot 0{,}5 = 1 - 0{,}125 = 0{,}825$$

R/S: $0{,}825$

10)



$AB = VA = 12$ cm

$A_1B_1 = 8$ cm

$$OC = \frac{AC}{2} = \frac{AB\sqrt{2}}{2} = 6\sqrt{2} \text{ cm}$$

$$O_1C_1 = \frac{A_1C_1}{2} = \frac{A_1B_1\sqrt{2}}{2} = 4\sqrt{2} \text{ cm}$$

Th. Pitagora în ΔVOC: $VO^2 = VC^2 - OC^2 = 144 - 72 = 72 \Rightarrow$
$\Rightarrow VO = 6\sqrt{2}$ cm

$$A_1C \parallel AC \Rightarrow \Delta VO_1C_1 \sim \Delta VOC \Rightarrow \frac{VO_1}{VO} = \frac{O_1C_1}{OC} \Rightarrow$$

$$\Rightarrow \frac{VO_1}{6\sqrt{2}} = \frac{4\sqrt{2}}{6\sqrt{2}} \Rightarrow VO_1 = 4\sqrt{2} \text{ cm} \Rightarrow O_1O = VO - VO_1 = 2\sqrt{2} \text{ cm}$$

$$V_{tr.} = \frac{1}{3} \cdot OO_1\left(AB^2 + A_1B_1^2 + AB \cdot A_1B_1\right) =$$

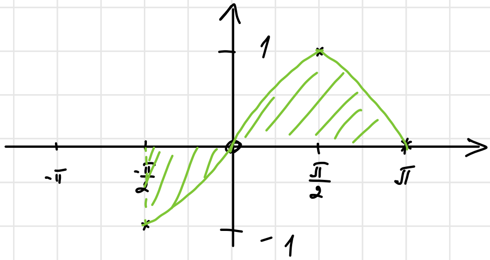
$$= \frac{1}{3} \cdot 2\sqrt{2} \cdot (144 + 64 + 96) = \frac{608\sqrt{2}}{3} \text{ (cm}^3\text{)}$$

R/S: $\frac{608\sqrt{2}}{3}$ cm^3

Testul 7

11) $\mathcal{A}(\Gamma_f) = 3 \cdot \int_0^{\frac{\pi}{2}} \sin x\,dx = -3\cos x \Big|_0^{\frac{\pi}{2}} =$

$= -3(0-1) = 3$ (un. p.)



R/s: 3 un. p.

12) $2mx^2 + 2(2m-1)x + 2m > 0,\ \forall x \in \mathbb{R}$, $m - ?$

a) $2m = 0 \Rightarrow m = 0 \Rightarrow -2x > 0,\ \forall x \in \mathbb{R}$ - Fals

b) $m \neq 0$

b_1) $m > 0$ (graficul f-ției asociate inecuației este o parabolă cu ramurile în sus)

$2mx^2 + 2(2m-1)x + 2m = 0 \,/ : 2$

$mx^2 + (2m-1)x + m = 0$

$\Delta = (2m-1)^2 - 4m^2 = 4m^2 - 4m + 1 - 4m^2 = 1 - 4m$

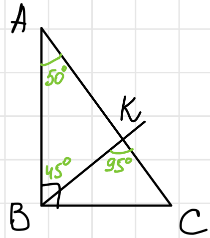
$\Delta < 0 \Rightarrow 1 - 4m < 0 \Rightarrow m > \frac{1}{4}$

b_2) Dacă $m < 0$ (graficul f-ției asociate este o parabolă cu ramurile în jos), atunci inecuația inițială nu poate avea $S = \mathbb{R}$.

R/s: $m \in \left(\frac{1}{4}; +\infty\right)$

Testul 8

1) = ; 2) > ; 3)



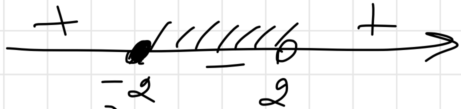
$m(\angle BKC) = 95^\circ$

4) $3^{2+\frac{1}{2}\log_3 16} = 3^2 \cdot 3^{\log_3\sqrt{16}} = 9\cdot 4 = 36$

R/s: 36

5) $\frac{2x}{x-2} \le 1 \Rightarrow \frac{2x-x+2}{x-2} \le 0 \Rightarrow \frac{x+2}{x-2} \le 0$

DVA $= \mathbb{R}\setminus\{2\}$



R/s: $S = [-2; 2)$

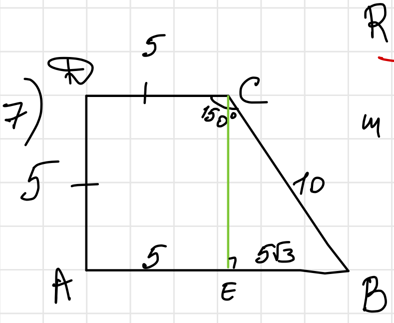
6) $3a + (5-2i)b = 1+2i$

$3a + 5b - 2bi = 1 + 2i$

$\begin{cases} 3a+5b = 1 \\ -2b = 2 \end{cases} \Leftrightarrow \begin{cases} 3a = 6 \\ b = -1 \end{cases} \Leftrightarrow \begin{cases} a = 2 \\ b = -1 \end{cases}$

R/s: $a = 2$; $b = -1$

7)



$m(\angle ECB) = 60^\circ \Rightarrow m(\angle B) = 30^\circ \Rightarrow CB = 2CE = 10\,cm$

$EB = CE\sqrt{3} = 5\sqrt{3}\,cm$

$P_{ABCD} = AB + BC + DC + AD = 5 + 5\sqrt{3} + 10 + 5 + 5 = (25 + 5\sqrt{3})\,(cm)$

R/S: $(25+5\sqrt{3})\,cm$

Testul 8

8) $F(x)=\int(\cos x-\sin x)dx=\sin x+\cos x+C$

$F\left(\frac{3\pi}{2}\right)+2=0 \Rightarrow -1+0+C+2=0 \Rightarrow C=-1$

$\sin x+\cos x-1=0 \Rightarrow \sin x+\cos x=1 \,/\cdot\frac{\sqrt{2}}{2} \Rightarrow \frac{\sqrt{2}}{2}\sin x+\frac{\sqrt{2}}{2}\cos x=\frac{\sqrt{2}}{2}\Rightarrow$

$\Rightarrow \sin x\cdot\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cdot\cos x=\frac{\sqrt{2}}{2}\Rightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}\Rightarrow$

$\Rightarrow x+\frac{\pi}{4}=(-1)^k\cdot\frac{\pi}{4}+k\pi,\ k\in\mathbb{Z} \Rightarrow x=-\frac{\pi}{4}+(-1)^k\cdot\frac{\pi}{4}+k\pi,\ k\in\mathbb{Z}$

$k=0\Rightarrow x=0\notin D(F)$; $\quad k=2\Rightarrow x=2\pi\in D(F)$

$k=1 \Rightarrow x=\frac{\pi}{2}\notin D(F)$;

R/s: $x=2\pi$

9) 3a/7r. $\xrightarrow{4\,b.}$ A={bilele extrase sunt de 2 culori}

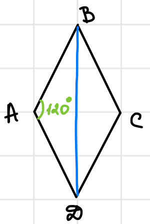
B={se extrag 4 b. albe}; C={se extrag 4 b. roșii}

$P(A)=1-P(B)-P(C)=1-0-\frac{C_7^4}{C_{10}^4}=1-\frac{35}{210}=1-\frac{1}{6}=\frac{5}{6}$

$C_7^4=\frac{7!}{4!\cdot 3!}=\frac{5\cdot6\cdot7}{2\cdot3}=35$; $C_{10}^4=\frac{10!}{6!\cdot4!}=\frac{7\cdot8\cdot9\cdot10}{2\cdot3\cdot4}=210$

R/s: $\frac{5}{6}$

10)



$\mathcal{A}_{ABCD}=AB^2\cdot\sin(\angle A)\Rightarrow 18\sqrt{3}=AB^2\cdot\frac{\sqrt{3}}{2}\Rightarrow AB^2=36\Rightarrow AB=6$ cm

Th. cosinusului în $\triangle ABD$:

$BD^2=AB^2+AD^2-2\cdot AB\cdot AD\cdot\cos(\angle A)$

$BD^2=36+36+2\cdot36\cdot\frac{1}{2}$

$BD^2=36\cdot3\Rightarrow BD=6\sqrt{3}$ cm

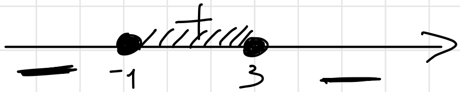
R/s: $6\sqrt{3}$ cm

Testul 8

11) $(2-|x-1|)\cdot \lg(4x^2+8)\geq 0$ $\quad$ DVA $=\mathbb{R}$

$(2-|x-1|)\cdot \lg(4x^2+8)=0$

$$\left[\begin{array}{l}2-|x-1|=0\\ \lg(4x^2+8)=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}|x-1|=2\\ 4x^2+8=1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x-1=2\\ x-1=-2\\ 4x^2+7=0\end{array}\right. \Rightarrow \left[\begin{array}{l}x=3\\ x=-1\end{array}\right.$$



R/s: $S=[-1;3]$

12) $f'(x)=5x^4$; $\quad t_1: y=5a^4(x-a)+a^5 \Rightarrow t_1: y=5a^4x-4a^5$

$g'(x)=6x^5$; $\quad t_2: y=6a^5(x-a)+a^6 \Rightarrow t_2: y=6a^5x-5a^6$

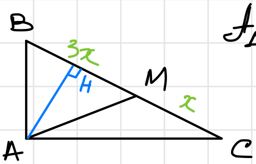
$$t_1\cap t_2=\emptyset \Rightarrow \begin{cases} t_1 \parallel t_2\\ t_1\neq t_2\end{cases} \Leftrightarrow \begin{cases}5a^4=6a^5\\ 4a^5\neq 5a^6\end{cases} \Leftrightarrow \begin{cases}6a^5-5a^4=0\\ 5a^6-4a^5\neq 0\end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases}a^4(6a-5)=0\\ a^5(5a-4)\neq 0\end{cases} \Leftrightarrow \begin{cases}\left[\begin{array}{l}a=0\\ a=\frac{5}{6}\end{array}\right.\\ a\neq 0\\ a\neq \frac{4}{5}\end{cases} \Rightarrow a=\frac{5}{6}$$

R/s: $a=\frac{5}{6}$

Testul 9

1) $0,4$; 2) $E(f)=[-5;1]$; 3)



$\mathcal{A}_{\Delta ABC}=\frac{1}{2}\cdot BC\cdot AH$

$12=\frac{1}{2}\cdot 4x\cdot AH$

$x\cdot AH=6$

$\mathcal{A}_{\Delta AMC}=\frac{1}{2}\cdot AH\cdot MC=$

$=\frac{1}{2}\cdot AH\cdot x=3(cm^2)$

$\underline{\mathcal{A}_{\Delta AMC}=3cm^2}$

4) $\left(\frac{1}{\sqrt{2}}\right)^{-2}+\log_{\frac{1}{2}}2=2-1=1$

$\underline{R/s: 1}$

5) $\log_{\frac{1}{2}}\left(x^2-4x+1\right)+2=0 \Rightarrow \log_{\frac{1}{2}}\left(x^2-4x+1\right)=-2$

DVA: $x^2-4x+1>0$

$x^2-4x+1=0$

$\Delta=16-4=12$

$x_1=2-\sqrt{3}$; $x_2=2+\sqrt{3}$

$+$ $\quad 2-\sqrt{3} \quad - \quad 2+\sqrt{3} \quad$ $+$

DVA $=(-\infty;2-\sqrt{3})\cup(2+\sqrt{3};+\infty)$

$x^2-4x+1=4$

$x^2-4x-3=0$

$\Delta=16+12=28$

$x_1=2+\sqrt{7}\in DVA$

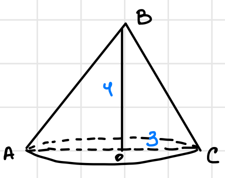
$x_2=2-\sqrt{7}\in DVA$

$\underline{R/s: S=\{2\pm\sqrt{7}\}}$

6) $(z+5)^2+4=0 \Rightarrow (z+5)^2=-4 \Rightarrow \left[\begin{array}{l} z+5=2i \\ z+5=-2i \end{array}\right. \Rightarrow \left[\begin{array}{l} z=2i-5 \\ z=-2i-5 \end{array}\right.$

R/s: $z\in\{\pm 2i-5\}$

7)



$\mathcal{A}_{\Delta ABC}=12cm^2 \Rightarrow \frac{1}{2}\cdot BO\cdot AC=12 \Rightarrow \frac{1}{2}\cdot H\cdot 2R=12 \Rightarrow HR=12 \Rightarrow R=3cm$

Th. Pitagora în ΔBOC: $BC^2=16+9=25 \Rightarrow BC=5cm$

$\mathcal{A}_{tot}=\mathcal{A}_{lat}+\mathcal{A}_{baz}=\pi R g+\pi R^2=\pi\cdot 3\cdot 5+\pi\cdot 9=24\pi(cm^2)$

$V_{con}=\frac{1}{3}\cdot\pi R^2 H=\frac{1}{3}\cdot 9\pi\cdot 4=12\pi\,(cm^3)$

$\underline{R/s: 24\pi\,cm^2;\ 12\pi\,cm^3}$

Testul 9

8) $$\int_1^e \frac{\ln x}{x^2}dx = \left| \begin{matrix} u=\ln x;\ du=\frac{1}{x}dx \\ dv=x^{-2}dx;\ v=-\frac{1}{x} \end{matrix} \right| = -\frac{\ln x}{x}\Big|_1^e + \int_1^e \frac{1}{x^2}dx =$$

$$=\left(-\frac{\ln x}{x}-\frac{1}{x}\right)\Big|_1^e = -\frac{1}{e}-\frac{1}{e}+1 = -\frac{2}{e}+1=\frac{e-2}{e}$$

$$\mathcal{A}(\Gamma_f)=\frac{e-2}{e}\text{ un. p.}$$

R/S: $\frac{e-2}{e}$ un. p.

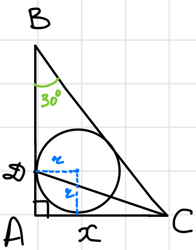
9) $A=\{$se iau 2 cărți de matem.$\}$; Notăm: x – nr. de cărți de matem.

$$P(A)=\frac{C_x^2}{C_{10}^2}=\frac{x!}{\cancel{2!}\cdot(x-2)!}\cdot\frac{8!\cdot\cancel{2!}}{10!}=\frac{x(x-1)}{90}$$

$$\frac{x(x-1)}{90}=\frac{1}{3} \Rightarrow x(x-1)=30 \Rightarrow x^2-x-30=0$$

$x_1=-5\notin\mathbb{N}$
$x_2=6$ $\Rightarrow x=6$

R/S: 6 cărți de matematică

10) 

$r=\sqrt{3}$ cm

$m(\angle B)=30^\circ \Rightarrow AC=x;\ BC=2x;\ AB=x\sqrt{3}.$

$r=\frac{AB+AC-BC}{2} \Rightarrow 2\sqrt{3}=x\sqrt{3}+x+2x \Rightarrow$

$\Rightarrow x(3+\sqrt{3})=2\sqrt{3} \Rightarrow \cancel{\sqrt{3}}x(\sqrt{3}+1)=2\cancel{\sqrt{3}} \Rightarrow x=\frac{\overset{\sqrt{3}-1}{\cancel{2}}}{\sqrt{3}+1}=\frac{2(\sqrt{3}-1)}{2}$

$x=\sqrt{3}-1 \Rightarrow AC=(\sqrt{3}-1)$ cm

Th. Pitagora în ΔADC: $CD^2=AD^2+AC^2=3+(\sqrt{3}-1)^2 \Rightarrow$
$\Rightarrow CD^2=3+3-2\sqrt{3}+1=7-2\sqrt{3} \Rightarrow CD=\sqrt{7-2\sqrt{3}}$ cm

R/S: $\sqrt{7-2\sqrt{3}}$ cm

Testul 9

11) $f'(x) = -\dfrac{1}{1-x} = \dfrac{1}{x-1}$; $g'(x) = +\dfrac{1}{(1-x)^2} = \dfrac{1}{(x-1)^2}$

$D(f) = (-\infty; 1)$

$D(g) = \mathbb{R} \setminus \{1\}$

$$f'(x) + g'(x) = 2 \Rightarrow \frac{1}{x-1} + \frac{1}{(x-1)^2} = 2 \Rightarrow \frac{x}{(x-1)^2} = 2 \Rightarrow$$

$$\Rightarrow 2x^2 - 4x + 2 = x \to 2x^2 - 5x + 2 = 0$$

$$\Delta = 25 - 16 = 9$$

$$x_1 = \frac{1}{2}$$

$$x_2 = 2 \notin D(f)$$

R/s: $S = \left\{\frac{1}{2}\right\}$

12) $(m^2 - 3m)\sin x = 4m - 12$, $S \neq \emptyset$

a) $m^2 - 3m = 0 \Rightarrow m(m-3) = 0 \Rightarrow \left[\begin{array}{l} m = 0 \\ m = 3 \end{array}\right.$

Dacă $m = 0 \Rightarrow 0 \cdot \sin x = -12 \Rightarrow S = \emptyset$

Dacă $m = 3 \Rightarrow 0 \cdot \sin x = 0 \Rightarrow S = \mathbb{R}$

b) $m \in \mathbb{R} \setminus \{0; 3\}$

$$\sin x = \frac{4m-12}{m^2 - 3m} \Rightarrow \sin x = \frac{4(m-3)}{m(m-3)} \Rightarrow \sin x = \frac{4}{m}$$

$$S \neq \emptyset \Rightarrow \begin{cases} \dfrac{4}{m} \leq 1 \\ \dfrac{4}{m} \geq -1 \end{cases} \Leftrightarrow \begin{cases} \dfrac{4-m}{m} \leq 0 \\ \dfrac{4+m}{m} \geq 0 \end{cases} \Leftrightarrow \begin{cases} m \in (-\infty; 0) \cup [4; +\infty) \\ m \in (-\infty; -4] \cup (0; +\infty) \end{cases}$$

R/s: $m \in (-\infty; -4] \cup [4; +\infty) \cup \{3\}$

Testul 10

1) $-3 \boxed{<} -2$; 2) $E \boxed{>} 0$; 3) 55^0 ; 4) $2^{\log_2 7 + \log_3 \frac{1}{3}} = 2^{\log_2 7} \cdot 2^{-1} = \frac{7}{2}$

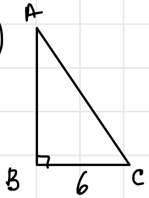
R/s: $\frac{7}{2}$.

5) $\left(\frac{2}{3}\right)^{5x^2} - \left(\frac{2}{3}\right)^{6-x^2} = 0 \Rightarrow \left(\frac{2}{3}\right)^{5x^2} = \left(\frac{2}{3}\right)^{6-x^2} \Rightarrow 5x^2 = 6 - x^2 \Rightarrow 6x^2 = 6 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

R/s: $S = \{\pm 1\}$

6) $A = B \Rightarrow \begin{cases} 2x + 3i = 4y + 3i \\ yi = i \end{cases} \Leftrightarrow \begin{cases} 2x = 4 \\ y = 1 \end{cases} \Leftrightarrow \begin{cases} x = 2 \\ y = 1 \end{cases}$

R/s: $x = 2; y = 1$

7) 

$BC = 6$; $R = 5\,cm \Rightarrow AC = 10\,cm$

Th. Pitagora în $\Delta ABC \Rightarrow AB^2 = 100 - 36 = 64 \Rightarrow AB = 8\,cm$

$\mathcal{A}_{\Delta ABC} = \frac{AB \cdot BC}{2} = \frac{8 \cdot 6}{2} = 24\ (cm^2)$

R/s: $24\,cm^2$

8) $f'(x) = \frac{\sqrt{3}}{3} e^{\frac{\sqrt{3}}{3}x - 2}$

Tang. formează cu direcția poz. a axei Ox un unghi de $30^0 \Rightarrow f'(x_0) = tg\, 30^0 \Rightarrow$

$\Rightarrow \frac{\sqrt{3}}{3} e^{\frac{\sqrt{3}}{3}x_0 - 2} = \frac{\sqrt{3}}{3} \Rightarrow e^{\frac{\sqrt{3}}{3}x_0 - 2} = 1 \Rightarrow \frac{\sqrt{3}}{3}x_0 - 2 = 0 \Rightarrow x_0 = \frac{2 \cdot 3}{\sqrt{3}} \Rightarrow x_0 = 2\sqrt{3}$

$f(2\sqrt{3}) = e^{\frac{\sqrt{3}}{3} \cdot 2\sqrt{3} - 2} = 1$

$y = \frac{\sqrt{3}}{3} \cdot (x - 2\sqrt{3}) + 1 \Rightarrow y = \frac{\sqrt{3}}{3}x - 2 + 1 \Rightarrow y = \frac{\sqrt{3}}{3}x - 1$

R/s: $y = \frac{\sqrt{3}}{3}x - 1$

9) $\left.\begin{matrix} 4b. \\ 5ch. \\ 3b. \end{matrix}\right\} \xrightarrow{4p.}$ $A = \{$se alege cel puțin câte un specialist de fiecare profil$\}$

$B = \{$se aleg 4 buc.$\}$; $C = \{$se aleg 4 chelneri$\}$

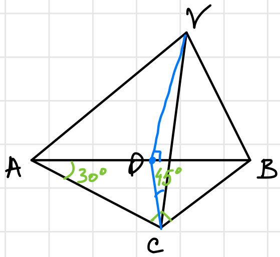
$P(A) = 1 - P(B) - P(C) = 1 - \frac{C_4^4}{C_{12}^4} - \frac{C_5^4}{C_{12}^4} = 1 - \frac{1+5}{495} = 1 - \frac{6^{/3}}{495} = 1 - \frac{2}{165} = \frac{163}{165}$

$C_{12}^4 = \frac{12!}{8! \cdot 4!} = \frac{9 \cdot 10 \cdot 11 \cdot 12}{2 \cdot 3 \cdot 4} = 495$

R/s: $\frac{163}{165}$

Testul 10

10)



Muchiile laterale formează cu planul bazei unghiuri congruente $\Rightarrow$ O - centrul cercului circumscris ΔABC.

$$R = \frac{AB}{2} = 10\,cm\ ;\ OC = R = 10\,cm$$

ΔVOC - isoscel $\Rightarrow VO = OC = 10\,cm$.

$$m(\angle A) = 30^\circ \Rightarrow BC = \frac{1}{2}AB = 10\,cm$$

$$AC = BC\sqrt{3} = 10\sqrt{3}\,cm.$$

$$V_{piz} = \frac{1}{3}\cdot\frac{AC\cdot BC}{2}\cdot VO = \frac{1}{3}\cdot\frac{10\sqrt{3}\cdot 10}{2}\cdot 10 = \frac{500\sqrt{3}}{3}(cm^3)$$

R/S: $\frac{500\sqrt{3}}{3}\,cm^3$

11) $$\int_0^1 \frac{4x-6}{x^2-3x+7}dx = \left| \begin{array}{l} x^2-3x+7=t \\ dt=(2x-3)dx \\ dx=\frac{dt}{2x-3} \\ x=0 \Rightarrow t=7 \\ x=1 \Rightarrow t=5 \end{array} \right| = \int_7^5 \frac{2(2x-3)}{t}\cdot\frac{dt}{2x-3} = \int_7^5 \frac{2}{t}dt =$$

$$= 2\ln|t|\Big|_7^5 = 2(\ln 5 - \ln 7) = 2\ln\frac{5}{7}$$

R/S: $2\ln\frac{5}{7}$

12) $x^2 - 2^{a+2}\cdot x - 2^{a+3} + 12 > 0,\ \forall x \in \mathbb{R}$

$\Delta < 0 \Rightarrow 2^{2a+4} - 4(2^{a+3}+12) < 0$

$2^{2a}\cdot 16 - 32\cdot 2^a - 48 < 0 \,/:16$

$2^{2a} - 2\cdot 2^a - 3 < 0$

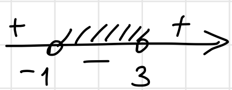
Notăm: $2^a = t,\ t > 0$

$t^2 - 2t - 3 < 0$

$t^2 - 2t - 3 = 0$

$t_1 = -1$

$t_2 = 3$



$t \in (-1;3) \Rightarrow 2^a \in (-1;3) \Rightarrow$

$\Rightarrow 2^a < 3 \Rightarrow a < \log_2 3$

R/S: $a \in (-\infty; \log_2 3)$

Testul 11

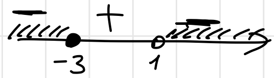
1) = ; 2) $f'(0)<0$; 3) 90^0 ; 4) $3^{2\log_9 0,5+1}-\sqrt[3]{0,125}=3^{2\cdot\frac{1}{2}\log_3 0,5}\cdot 3-0,5=$
$=0,5\cdot 3-0,5=1,5-0,5=1$

R/s: 1

5) $d=-2-2=-4$; $\frac{-4}{x-1}\le 1 \Rightarrow \frac{-4-x+1}{x-1}\le 0 \Rightarrow \frac{-3-x}{x-1}\le 0$

$DVA=\mathbb{R}\setminus\{1\}$

$-3-x=0$
$x=-3$



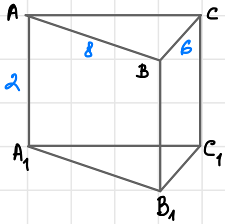
R/s: $S=(-\infty;-3]\cup(1;+\infty)$

6) $a+(5-2i)b=(1+2i)^2 \Rightarrow a+5b-2bi=-3+4i \Rightarrow \begin{cases} a+5b=-3 \\ -2b=4 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} a=7 \\ b=-2 \end{cases}$

R/s: $a=7$; $b=-2$

7)



$\mathcal{A}_{\Delta ABC}=\frac{AB\cdot BC}{2} \Rightarrow \frac{8\cdot BC}{2}=24 \Rightarrow BC=6$ cm

$V_{pris.}=\mathcal{A}_{\Delta ABC}\cdot A_1A \Rightarrow 48=24\cdot A_1A \Rightarrow A_1A=2$ cm

Th. lui Pitagora în ΔABC: $AC^2=64+36=100 \Rightarrow AC=10$ cm

Th. lui Pitagora în ΔAA_1C: $A_1C^2=100+4=104 \Rightarrow A_1C=2\sqrt{26}$ cm

R/s: $2\sqrt{26}$ cm

8) $D(f)=\mathbb{R}\setminus\{0\}$

$f'(x)=2x+\frac{54}{x^2}$; $f'(x)=0 \Rightarrow 2x+\frac{54}{x^2}=0 \Rightarrow 2x=-\frac{54}{x^2} \Rightarrow 2x^3=-54 \Rightarrow$

$\Rightarrow x^3=-27 \Rightarrow x=-3$

x	$-\infty$		-3		0		$+\infty$
f'		$-$		$+$	⋮	$+$	
f					⋮		

R/s: $f \nearrow$ pentru $x\in[-3;0)\cup(0;+\infty)$

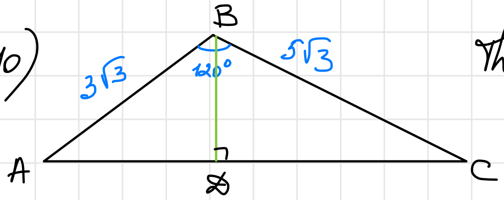
9) $\left.\begin{matrix} 6b. \\ 4f. \end{matrix}\right\} \xrightarrow{4el.} A=\{$se aleg și fete$\}$

$P(A)=1-P(\bar{A})=1-\frac{C_6^4}{C_{10}^4}=1-\frac{6!}{4!\cdot 2!}\cdot\frac{4!\cdot 6!}{10!}=1-\frac{1}{14}=\frac{13}{14}$

R/s: $\frac{13}{14}$

Testul 11

10)



Th. cosinusului în ΔABC:

$AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos(\angle B)$

$AC^2 = 27 + 75 + 2 \cdot 3\sqrt{3} \cdot 5\sqrt{3} \cdot \frac{1}{2}$

$AC^2 = 102 + 45 \Rightarrow AC^2 = 147 \Rightarrow AC = 7\sqrt{3}$ cm

Cea mai mică înălțime corespunde celei mai mari laturi $\Rightarrow AC$

$$\mathcal{A}_{\Delta ABC} = \frac{AB \cdot BC \cdot \sin(\angle B)}{2} = \frac{3\sqrt{3} \cdot 5\sqrt{3} \cdot \sqrt{3}}{4} = \frac{45\sqrt{3}}{4} \ (cm^2)$$

$$\mathcal{A}_{\Delta ABC} = \frac{1}{2} BD \cdot AC \Rightarrow \frac{45\sqrt{3}}{4} = \frac{1}{2} \cdot BD \cdot 7\sqrt{3} \Rightarrow BD = \frac{45}{14} \text{ cm}$$

R/S: $\frac{45}{14}$ cm

11) $$\int_0^{\frac{\pi}{2}} (\sin x + \cos 2x)\,dx = \left(-\cos x + \frac{1}{2}\sin 2x\right)\Big|_0^{\frac{\pi}{2}} = 0 + 0 + 1 - 0 = 1$$

R/S: 1

12) $(x^2 - 4x - 5)\sqrt{3^x - m} = 0$, $|S| = 2$

DVA: $3^x - m \geq 0 \Rightarrow 3^x \geq m \Rightarrow x \geq \log_3 m$

$$\left[\begin{array}{l} x^2 - 4x - 5 = 0 \\ \sqrt{3^x - m} = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = 5 \\ x = -1 \\ \sqrt{3^x - m} = 0 \end{array}\right. \overset{|S| = 2}{\Rightarrow} \text{ ec. } \sqrt{3^x - m} = 0 \text{ nu are soluții} \Rightarrow$$

$\Rightarrow m \leq 0$

R/S: $m \in (-\infty; 0]$

Testul 12

1) -2; 2) convexă; 3) 4;

4) $\frac{1}{2}\log_{\sqrt{5}}25-\log_5 125-2=\frac{1}{2}\cdot 2\cdot\log_5 25-3-2=2-3-2=-3$

R/s: -3

5) $d=6-2+2+2=8$

$\frac{x-8}{x}\le 8 \Rightarrow \frac{x-8x-8x}{x}\le 0 \Rightarrow \frac{-7x-8}{x}\le 0$; DVA $=\mathbb{R}\setminus\{0\}$

$-7x-8=0 \Rightarrow x=-\frac{8}{7}$

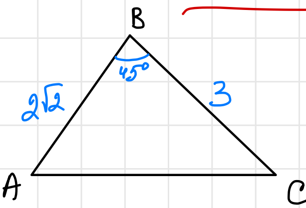
$-\frac{8}{7}$ $\quad$ $+$ $\quad$ 0

R/s: $S=\left(-\infty;-\frac{8}{7}\right]\cup(0;+\infty)$

6) $(z+4-i)^2-16=0 \Rightarrow (z+4-i)^2=16 \Rightarrow \left[\begin{array}{l} z+4-i=4 \\ z+4-i=-4 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} z=i \\ z=-8+i \end{array}\right.$

R/s: $z\in\{-8+i;\ i\}$

7)



$\mathcal{A}_{\Delta ABC}=\frac{1}{2}\cdot AB\cdot BC\cdot\sin(\sphericalangle B)$

$3=\frac{1}{2}\cdot 2\sqrt{2}\cdot 3\cdot\sin(\sphericalangle B) \Rightarrow \sin(\sphericalangle B)=\frac{1}{\sqrt{2}} \Rightarrow$

$\Rightarrow m(\sphericalangle B)=45^\circ$

Th. cosinusului în ΔABC: $AC^2=AB^2+BC^2-2AB\cdot BC\cdot\cos(\sphericalangle B)$

$AC^2=8+9-2\cdot\cancel{2}\sqrt{2}\cdot 3\cdot\frac{\sqrt{2}}{\cancel{2}} \Rightarrow AC^2=17-12 \Rightarrow AC^2=5$

$AC=\sqrt{5}$ cm

R/s: $AC=\sqrt{5}$ cm

8) $f'(x)=4e^{\frac{x}{2}}+2x\cdot e^{\frac{x}{2}}$; $f''(x)=2e^{\frac{x}{2}}+2\cdot e^{\frac{x}{2}}+x\cdot e^{\frac{x}{2}}=4e^{\frac{x}{2}}+xe^{\frac{x}{2}}=$
$=e^{\frac{x}{2}}(4+x)$

$f''(x)=0 \Rightarrow e^{\frac{x}{2}}(4+x)=0 \Rightarrow x=-4$

x	$-\infty$		-4		$+\infty$
f'		$-$		$+$	
f		⌒		⌣	

R/s: f este convexă p/u $x\in(-4;+\infty)$

Testul 12

9) $\checkmark \quad \checkmark \quad \checkmark \quad \times$

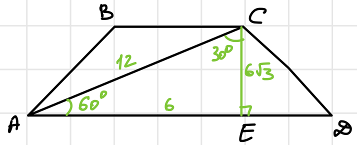
$\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{3}{4}$

$A = \{$din 4 aruncări, va nimeri exact de 3 ori$\}$

$$P(A) = \cancel{4}^{1} \cdot \frac{1}{\cancel{4}_{1}} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{64}$$

R/S: $\frac{3}{64}$

10)



$m(\angle ACE) = 30^\circ \Rightarrow AE = \frac{1}{2}AC = 6\,cm$

$CE = AE\sqrt{3} = 6\sqrt{3}\,cm$

11) $\cos x = -\frac{4}{5} \Rightarrow \sin x = -\sqrt{1-\cos^2 x} = -\sqrt{1-\frac{16}{25}} = -\frac{3}{5} \Rightarrow tg\, x = \frac{3}{4}$

$$E(x) = \frac{9}{25} + 2\cdot\frac{3}{4} + \frac{5}{4} = \frac{9}{25} + \frac{11}{4} = \frac{36+275}{100} = \frac{311}{100}$$

R/S: $\frac{311}{100}$

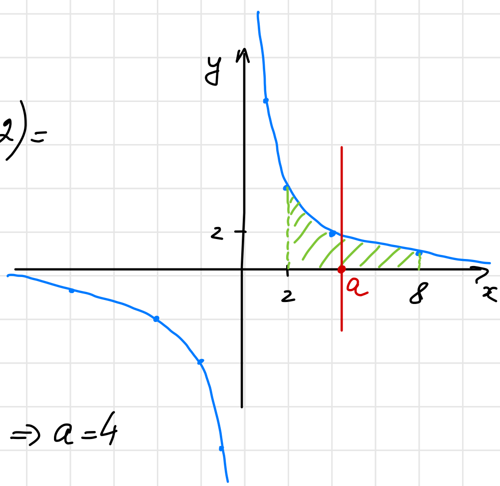
12)

$$\mathcal{I}_1 = \int_2^8 \frac{8}{x}\,dx = 8\ln|x|\Big|_2^8 = 8(\ln 8 - \ln 2) =$$

$$= 8\ln 4 = 16\ln 2$$

$$\mathcal{I}_2 = \int_2^a \frac{8}{x}\,dx = 8\ln|x|\Big|_2^a = 8(\ln a - \ln 2) =$$

$$= 8\ln\frac{a}{2}.$$



$$\mathcal{I}_1 = 2\mathcal{I}_2 \Rightarrow 16\ln 2 = 16\ln\frac{a}{2} \Rightarrow \frac{a}{2} = 2 \Rightarrow a = 4$$

R/S: $a = 4$

Testul 13

1) -5; 2) „strict descrescătoare"; 3) 6 cm

4) $36^{\log_6 5} - 3^{\log_9 36} + \sqrt[3]{8} = \left(6^{\log_6 5}\right)^2 - 3^{2\cdot\frac{1}{2}\log_3 6} + 2 = 25 - 6 + 2 = 21$

R/S: 21

5) $\frac{2}{\sqrt{2x+10}} \geq 1 \Rightarrow \frac{2-\sqrt{2x+10}}{\sqrt{2x+10}} \geq 0 \Rightarrow 2-\sqrt{2x+10} \geq 0 \Rightarrow \sqrt{2x+10} \leq 2 \Rightarrow$

$\Rightarrow 2x+10 \leq 4 \Rightarrow x \leq -3$

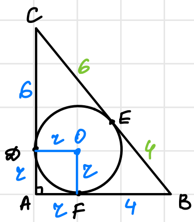
DVA $= (-5; +\infty)$

R/S: $S = (-5; -3]$

6) $(z-5i)^2 - 16 = 0 \Rightarrow (z-5i)^2 = 16 \Rightarrow \left[\begin{array}{l} z-5i=4 \\ z-5i=-4 \end{array}\right. \Rightarrow \left[\begin{array}{l} z=5i+4 \\ z=5i-4 \end{array}\right.$

R/S: $z \in \{5i \pm 4\}$

7)



$FB = EB = 4$ cm; $CD = CE = 6$ cm $\Rightarrow BC = 10$ cm; $AB = r+4$; $AC = 6+r$

Th. Pitagora în ΔABC: $(6+r)^2 + (4+r)^2 = 100 \Rightarrow 36+12r+r^2+16+8r+r^2 = 100 \Rightarrow$

$\Rightarrow 2r^2 + 20r - 48 = 0 \ /:2$

$r^2 + 10r - 24 = 0$

$r_1 = -12 < 0$

$r_2 = 2$

$\Rightarrow r = 2\text{cm} \Rightarrow AB = 6$ cm, $AC = 8$ cm

$\mathcal{A}_{\Delta ABC} = \frac{AB \cdot AC}{2} = \frac{6 \cdot 8}{2} = 24\,(\text{cm}^2)$

R/S: 24 cm^2

8) $\int_2^4 \left(2|x-3| + (x+1)^2\right)dx = 2\int_2^4 |x-3|\,dx + \int_2^4 (x+1)^2 dx = 2\left(\int_2^3 (3-x)\,dx + \int_3^4 (x-3)\,dx\right) + \int_2^4 (x+1)^2 dx =$

$= 2\left(\left(3x - \frac{x^2}{2}\right)\Big|_2^3 + \left(\frac{x^2}{2} - 3x\right)\Big|_3^4\right) + \frac{(x+1)^3}{3}\Big|_2^4 = 2\left(9 - \frac{9}{2} - 6 + 2 + 8 - 12 - \frac{9}{2} + 9\right) +$

$+ \frac{125}{3} - 9 = -7 + \frac{125}{3} = \frac{104}{3}$

R/S: $\frac{104}{3}$

Testul 13

9)

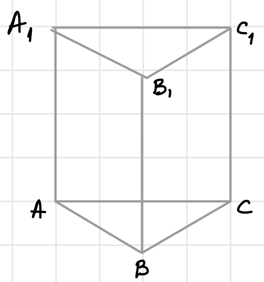
✓	✓	✓	x
$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{3}{5}$

$A = \{$din 4 lovituri, nimerește exact de 3 ori$\}$

$$P(A) = 4 \cdot \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{3}{5} = \frac{96}{625}$$

R/s: $\frac{96}{625}$

10)



$AB = A_1A = 12\ cm$

$$V_{pr.} = \mathcal{A}_{ABC} \cdot A_1A = \frac{AB^2\sqrt{3}}{4} \cdot A_1A = \frac{AB^3\sqrt{3}}{4} = \frac{12^3\sqrt{3}}{4} = 432\sqrt{3}\,(cm^3)$$

R/s: $432\sqrt{3}\ cm^3$

11) $f'(x) = \frac{1}{\sqrt{3}}(3x^2-4)$; Tang. formează cu Ox un unghi de măsura $\frac{5\pi}{6} \Rightarrow f'(x_0) = tg\,\frac{5\pi}{6} \Rightarrow$

$$\Rightarrow \frac{1}{\sqrt{3}}(3x_0^2-4) = -\frac{1}{\sqrt{3}} \Rightarrow 3x_0^2-4=-1 \Rightarrow 3x_0^2-3=0 \Rightarrow x_0^2-1=0 \Rightarrow \left[\begin{array}{l} x_0=1 \\ x_0=-1 \end{array}\right. \Rightarrow A(1;-\sqrt{3});\ B(-1;\sqrt{3})$$

R/s: $A(1;-\sqrt{3})$; $B(-1;\sqrt{3})$

12) $m\,e^{2x} - (3m-1)\,e^x + 2m - 1 = 0$; $\quad |S| = 2$

a) $m=0 \Rightarrow e^x - 1 = 0 \Rightarrow x = 0 \Rightarrow |S| = 1$

b) $m \neq 0 \Rightarrow m\,e^{2x} - (3m-1)\,e^x + 2m-1 = 0$

Notăm: $e^x = t,\ t>0$

$$m\cdot t^2 - (3m-1)\cdot t + 2m - 1 = 0$$

$$\Delta = (3m-1)^2 - 4m(2m-1)$$

$$\Delta = 9m^2 - 6m + 1 - 8m^2 + 4m$$

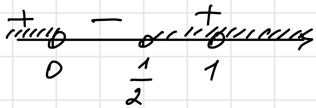
$$\Delta = m^2 - 2m + 1 = (m-1)^2$$

$|S| = 2 \Rightarrow \Delta > 0 \Rightarrow m \neq 1.$

$$t_1 = \frac{3m-1+m-1}{2m} = \frac{4m-2}{2m} = \frac{2m-1}{m}$$

$$t_2 = \frac{3m-1-m+1}{2m} = \frac{2m}{2m} = 1 > 0$$

$|S| = 2 \Rightarrow \frac{2m-1}{m} > 0 \Rightarrow (2m-1)\cdot m > 0$



R/s: $m \in (-\infty;0) \cup \left(\frac{1}{2};+\infty\right) \setminus \{1\}$

Testul 14

1) $<$; 2) $[4;+\infty)$; 3) 24 cm

4) $\sqrt{225}+\left(\frac{1}{2}\right)^3\cdot\sqrt{64}-\frac{7}{3}:\frac{3}{4}=15+\frac{8}{8}-\frac{28}{9}=16-\frac{28}{9}=\frac{116}{9}$

R/s: $\frac{116}{9}$

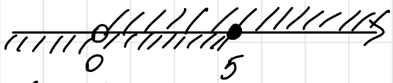
5) $-4-(2+2i)(z+3i)=0 \Rightarrow (2+2i)(z+3i)=-4 \Rightarrow z+3i=\frac{-4}{2+2i} \Rightarrow z+3i=\frac{-4(2-2i)}{8} \Rightarrow$

$\Rightarrow z+3i=\frac{-8(1-i)}{8} \Rightarrow z+3i=-1+i \Rightarrow z=-1-2i$

R/s: $S=\{-1-2i\}$

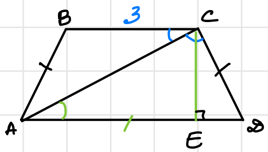
6) $\frac{5-x}{\sqrt{x}}\geq 0 \Rightarrow 5-x\geq 0 \Rightarrow x\leq 5$

DVA $=(0;+\infty)$



R/s: $S=(0;5]$

7)



$P_{ABCD}=42$ cm

$AD \parallel BC \Rightarrow \angle CAD \equiv \angle BCA$

AC - bisectoarea $\angle BCD \Rightarrow \angle BCA \equiv \angle ACD$

$\Rightarrow \Delta ACD$ - isoscel, $AD=CD$.

$P_{ABCD}=3AD+3 \Rightarrow 3AD+3=42 \Rightarrow 3AD=39 \Rightarrow AD=13$ cm

ABCD - isoscel $\Rightarrow CE=\frac{AD-BC}{2}=\frac{13-3}{2}=5$ cm

Th. Pitagora în Δ CED: $CE^2=169-25=144 \Rightarrow CE=12$ cm.

$\mathcal{A}_{ABCD}=\frac{AD+BC}{2}\cdot CE=\frac{13+3}{2}\cdot 12=96\ (cm^2)$

R/s: 96 cm^2

8) $\mathcal{A}(\Gamma_f)=\int_0^{\frac{1}{3}} e^{1-3x}dx=-\frac{1}{3}e^{1-3x}\Big|_0^{\frac{1}{3}}=-\frac{1}{3}(1-e)=\frac{e-1}{3}$ (un. p.)

R/s: $\frac{e-1}{3}$ un. p.

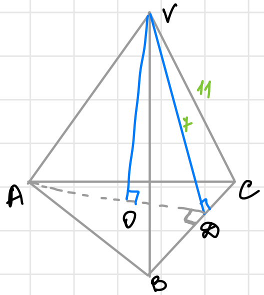
9) $\left.\begin{matrix}4f.\\6b.\end{matrix}\right\}\xrightarrow{4p.}$ $A=\{$se formează o echipă din 2 f. și 2 b.$\}$

$P(A)=\frac{C_4^2\cdot C_6^2}{C_{10}^4}=\frac{4!}{2!\cdot 2!}\cdot\frac{4!}{2!\cdot 2!}\cdot\frac{6!\cdot 4!}{10!}=\frac{6}{35}$

R/s: $\frac{6}{35}$

Testul 14

10)



Th. Pitagora în ΔVDC: $DC^2 = 121 - 49 = 72 \Rightarrow DC = 6\sqrt{3}$ cm

ΔABC – echilateral $\Rightarrow BC = 2DC = 12\sqrt{3}$ cm

$AD = \frac{BC\sqrt{3}}{2} = \frac{12\sqrt{3}\cdot\sqrt{3}}{2} = 18$ (cm)

$OD = \frac{1}{3}\cdot AD = \frac{18}{3} = 6$ (cm)

Th. lui Pitagora în ΔVOD: $VO^2 = 49 - 36 = 13 \Rightarrow VO = \sqrt{13}$ cm

$\mathcal{A}_{\Delta AVD} = \frac{1}{2}\cdot VO \cdot DA = \frac{1}{2}\cdot\sqrt{13}\cdot 18 = 9\sqrt{13}$ (cm^2)

R/s: $9\sqrt{13}$ cm^2

11) $f'(x) = 6x e^{x^2}$; $E(x) = 6xe^{x^2} - 6xe^{x^2} - \frac{1}{3}\cdot 3 - 0 = -1,\ \forall x \in \mathbb{R}$.

$E(1) = -1$

R/s: $E(1) = -1$

12) $\log_3(9^x + 9a^3) = x$, $|S| = 2$

$9^x + 9a^3 = 3^x$

$3^{2x} - 3^x + 9a^3 = 0$

Notăm: $3^x = t$, $t > 0$

$t^2 - t + 9a^3 = 0$

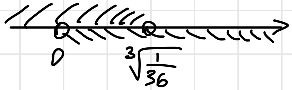
$\Delta = 1 - 36a^3$

$t_1 = \frac{1 - \sqrt{1-36a^3}}{2}$

$t_2 = \frac{1 + \sqrt{1-36a^3}}{2}$

$|S| = 2 \Rightarrow \begin{cases} \Delta > 0 \\ t_1 > 0 \end{cases} \Rightarrow \begin{cases} 1 - 36a^3 > 0 \\ \frac{1 - \sqrt{1-36a^3}}{2} > 0 \end{cases} \Rightarrow \begin{cases} 36a^3 < 1 \\ 1 - \sqrt{1-36a^3} > 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} a < \sqrt[3]{\frac{1}{36}} \\ \sqrt{1-36a^3} < 1 \end{cases} \Rightarrow \begin{cases} a < \sqrt[3]{\frac{1}{36}} \\ 1 - 36a^3 < 1 \end{cases} \Rightarrow \begin{cases} a < \sqrt[3]{\frac{1}{36}} \\ a > 0 \end{cases}$



R/s: $a \in \left(0; \sqrt[3]{\frac{1}{36}}\right)$

Testul 15

1) 7 ; 2) $\frac{1}{2}$; 3) 10 cm ; 4) $\log_2 \frac{1}{4} - \sqrt[3]{-27} = -2 + 3 = 1$

R/s: 1

5) $(0{,}25)^{2-x} \le \frac{1}{2^{x+3}} \Rightarrow \left(\frac{1}{4}\right)^{2-x} \le \left(\frac{1}{2}\right)^{x+3} \Rightarrow \left(\frac{1}{2}\right)^{4-2x} \le \left(\frac{1}{2}\right)^{x+3} \Rightarrow 4-2x \ge x+3 \Rightarrow$

$\Rightarrow -3x \ge -1 \Rightarrow x \le \frac{1}{3}$

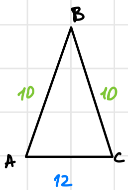
R/s: $S = \left(-\infty; \frac{1}{3}\right]$

6) $z(2+i)^2 = 4-3i \Rightarrow z = \frac{4-3i}{(2+i)^2} \Rightarrow z = \frac{4-3i}{4+4i-1} \Rightarrow z = {}^{3-4i)}\frac{4-3i}{3+4i} \Rightarrow z = \frac{12-9i-16i-12}{9+16} \Rightarrow$

$z = -\frac{25i}{25} \Rightarrow z = -i$

R/s: $z = -i$

7)



$P_{ABC} = 32\text{ cm} \Rightarrow AB = \frac{1}{2}(P_{ABC} - AC) = \frac{1}{2}(32-12) = 10(\text{cm})$

$p_{\Delta ABC} = 16\text{ cm}; \ \mathcal{A}_{\Delta ABC} = \sqrt{16 \cdot 6 \cdot 6 \cdot 4} = 48(\text{cm}^2)$

R/s: 48 cm^2

8) $f'(x) = 4e^{\frac{x}{2}} + 2x\,e^{\frac{x}{2}} = e^{\frac{x}{2}}(4+2x)$;

$f'(x) = 0 \Rightarrow e^{\frac{x}{2}}(4+2x) = 0 \Rightarrow 4+2x = 0 \Rightarrow x = -2$

x	$-\infty$	-2	$+\infty$
f'	$-$		$+$
f	$\searrow$		$\nearrow$

R/s: $f \nearrow$ p/u $x \in [-2; +\infty)$; $f \searrow$ p/u $x \in (-\infty; -2]$

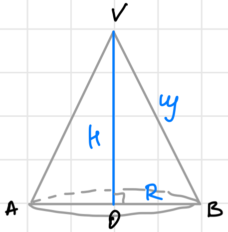
9) $\left.\begin{matrix} 5\text{ p} \\ 7\text{ f.p.} \end{matrix}\right\} \xrightarrow{12c.}$ $A = \{$se deschide cel puțin o cutie cu premiu$\}$

$P(A) = 1 - P(\bar{A}) = 1 - \frac{C_7^2}{C_{12}^2} = 1 - \frac{7!}{5! \cdot 2!} \cdot \frac{10! \cdot 2!}{12!} = 1 - \frac{7}{22} = \frac{15}{22}$

R/s: $\frac{15}{22}$

Testul 15

10)



$\frac{g}{R} = \frac{2}{1} \Rightarrow g = 2R$; $A_{lat.} = 72\pi \Rightarrow \pi R g = 72\pi \Rightarrow 2R^2 = 72 \Rightarrow$

$\Rightarrow R^2 = 36 \Rightarrow R = 6\text{ cm} \Rightarrow g = 12\text{ cm}$

$H^2 + R^2 = g^2 \Rightarrow H^2 = 144 - 36 = 108 \Rightarrow H = 6\sqrt{3}\text{ cm}$

$V_{con} = \frac{\pi R^2 H}{3} = \frac{\pi \cdot 36 \cdot 6\sqrt{3}}{3} = 72\sqrt{3}\pi (\text{cm}^3)$

R/S: $72\sqrt{3}\pi\text{ cm}^3$

11) $F(x) = 2x + 7\ln|x| + C$; $A(1;3) \in G_F \Rightarrow F(1) = 3 \Rightarrow 2 + C = 3 \Rightarrow C = 1 \Rightarrow F(x) = 2x + 7\ln|x| + 1 \Rightarrow F(e) = 2e + 1$

R/S: $F(e) = 2e + 1$

12) $(1-a^2)\sin x = a - 1$, $S \neq \emptyset$

a) $1 - a^2 = 0 \Rightarrow \left[\begin{array}{l} a = 1 \\ a = -1 \end{array}\right.$

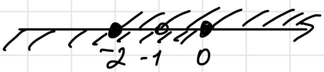
Dacă $a = 1 \Rightarrow 0 \cdot \sin x = 0 \Rightarrow S = \mathbb{R}$

Dacă $a = -1 \Rightarrow 0 \cdot \sin x = -2 \Rightarrow S = \emptyset$

b) $a \neq \pm 1 \Rightarrow \sin x = \frac{a-1}{1-a^2} \Rightarrow \sin x = \frac{a-1}{(1-a)(1+a)} \Rightarrow$

$\Rightarrow \sin x = -1 - a$

$S \neq \emptyset \Rightarrow \begin{cases} -1-a \leq 1 \\ -1-a \geq -1 \end{cases} \Leftrightarrow \begin{cases} -a \leq 2 \\ -a \geq 0 \end{cases} \Leftrightarrow \begin{cases} a \geq -2 \\ a \leq 0 \end{cases}$



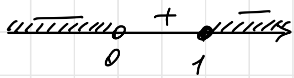
R/S: $a \in [-2;-1) \cup (-1;0] \cup \{1\}$

Testul 16

1) $-2; -1$; 2) 1 cm; 3) 2; 4) $\frac{1}{\sqrt[3]{125}} - \left(\frac{5}{2}\right)^{-1} = \frac{1}{5} - \frac{2}{5} = -\frac{1}{5}$

5) $d = 2 - 2 + 1 = 1$; $\frac{1-x}{x} \le 0$; $1 - x = 0 \Rightarrow x = 1$

DVA $= \mathbb{R}\setminus\{0\}$



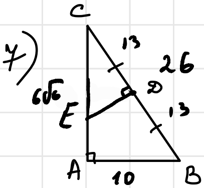
R/s: $S = (-\infty; 0) \cup [1; +\infty)$

6) $(2+i)x - (2-i)y = x - y + 2i \Rightarrow 2x + ix - 2y + iy = x - y + 2i \Rightarrow \begin{cases} 2x - 2y = x - y \\ x + y = 2 \end{cases} \Leftrightarrow \begin{cases} x = y \\ x + y = 2 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}$

R/s: $x = 1; y = 1$

7)



CTP. $AC = \sqrt{26^2 - 10^2} = \sqrt{(26-10)(26+10)} = \sqrt{6 \cdot 36} = 6\sqrt{6}$ cm

$CD = DB = 13$ cm. $\Delta CDE \sim \Delta CAB$

$\frac{CD}{CA} = \frac{DE}{AB} = \frac{CE}{CB}$; $\frac{13}{6\sqrt{3}} = \frac{DE}{10} = \frac{CE}{26} \Rightarrow$ $DE = \frac{65}{3\sqrt{3}}$, $CE = \frac{169}{3\sqrt{3}}$

$P_{\Delta CDE} = 13 + \frac{65}{3\sqrt{3}} + \frac{169}{3\sqrt{3}} = 13 + \frac{78}{\sqrt{3}} = (13 + 26\sqrt{3})$ cm

8) $A(x_0; f(x_0))$ – punct de tangență; $A = G_f \cap Oy \Rightarrow A(0; f(0))$

$f'(x) = 2e^{\frac{x}{2}}$; $f'(0) = 2$; $f(0) = 4$; $y = 2x + 4$

R/s: $y = 2x + 4$

9) $A = \{$se alege un nr. natural de 4 cifre pare$\}$

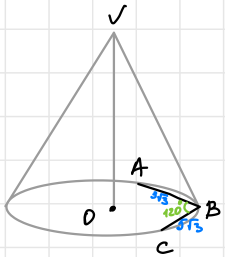
2,4,6,8   0,2,4,6,8   0,2,4,6,8   0,2,4,6,8

$P(A) = \frac{4 \cdot 5 \cdot 5 \cdot 5}{9 \cdot 10 \cdot 10 \cdot 10} = \frac{1}{18}$

R/s: $\frac{1}{18}$

Testul 16

10)



$g = 11$ cm

$\mathcal{A}_{\Delta ABC} = \frac{1}{2} AB \cdot BC \cdot \sin(\angle B) = \frac{1}{2} \cdot 3\sqrt{3} \cdot 5\sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{45\sqrt{3}}{4} \, (cm^2)$

Th. cosinusului în ΔABC: $AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos(\angle B) \Rightarrow$

$\Rightarrow AC^2 = 27 + 75 + 2 \cdot 3\sqrt{3} \cdot 5\sqrt{3} \cdot \frac{1}{2} \Rightarrow AC^2 = 147 \Rightarrow AC = 7\sqrt{3}$ cm

$$R = \frac{AB \cdot AC \cdot BC}{4 \cdot A_{\Delta ABC}} = \frac{3\sqrt{3} \cdot 5\sqrt{3} \cdot 7\sqrt{3}}{4 \cdot \frac{45\sqrt{3}}{4}} = 7 \, (cm)$$

$H^2 + R^2 = g^2 \Rightarrow H^2 = 121 - 49 = 72 \Rightarrow H = 6\sqrt{2}$ cm

$V_{con} = \frac{1}{3} \cdot \pi R^2 H = \frac{1}{3} \pi \cdot 49 \cdot 6\sqrt{2} = 98\pi\sqrt{2} \, (cm^3)$

R/s: $98\pi\sqrt{2}$ cm^3

11) $\alpha \in \left(-\pi; -\frac{\pi}{2}\right) \Rightarrow \cos\alpha < 0, \ \sin\alpha < 0$

$\sin\alpha = -\sqrt{1-\cos^2\alpha} = -\sqrt{1-\frac{16}{25}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}; \quad tg\,\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{3}{5} \cdot \frac{5}{4} = \frac{3}{4}$

$E(\alpha) = \frac{5}{12} \cdot 2\sin\alpha\cos\alpha + \frac{4}{5} tg\,\alpha = \frac{5}{12} \cdot 2 \cdot \frac{3}{5} \cdot \frac{4}{5} + \frac{4}{5} \cdot \frac{3}{4} = \frac{2}{5} + \frac{3}{5} = 1$

R/s: $E(\alpha) = 1$

12) $\frac{(2^x - a)(x-2)}{1-x} = 0 \qquad card\, S = 1$

$$\begin{cases} 1 - x \neq 0 \\ \left[\begin{array}{l} 2^x = a \\ x - 2 = 0 \end{array}\right. \end{cases} \Leftrightarrow \begin{cases} x \neq 1 \\ \left[\begin{array}{l} 2^x = a \\ x = 2 \end{array}\right. \end{cases} \overset{card\, S = 1}{\Rightarrow} \left[\begin{array}{l} a = 2 \\ a = 4 \\ a \leq 0 \end{array}\right. \Rightarrow a \in (-\infty; 0] \cup \{2; 4\}$$

R/s: $a \in (-\infty; 0] \cup \{2; 4\}$.

Test 17.

(1) $\log_2 \frac{1}{5} \boxed{<} \log_2 \frac{1}{2}$ $\Big|$ $\log_2 5^{-1} < \log_2 2^{-1}$

$-\log_2 5 < -\log_2 2$

(2) $\boxed{<}$ (3) $m(\angle BAC) = 80^\circ$.

(4) $\sqrt[3]{64} - \sqrt[4]{256} - \left(\frac{1}{2}\right)^{-2} = \sqrt[3]{4^3} - \sqrt[4]{4^4} - 2^2 = 4 - 4 - 4 = -4$

(5) $\begin{vmatrix} 2^{x-1} & 4 \\ \sqrt{2} & 4^x \end{vmatrix} \leq 0$ $2^{x-1} \cdot 4^x - 4\sqrt{2} \leq 0$

$2^{x-1} \cdot 2^{2x} \leq 2^2 \cdot 2^{\frac{1}{2}}$

$x - 1 + 2x \leq 2 + \frac{1}{2} \;\Big|\cdot 2 \Leftrightarrow 2x - 2 + 4x \leq 4 + 1 \Leftrightarrow 6x \leq 7$

$\Leftrightarrow x \leq \frac{7}{6}$ $\frac{7}{6}$ $S = (-\infty; 7/6]$

(6) $(2x - i)(3 - 3i) = x - (y + 1)i$

$6x - 6xi - 3i + 3i^2 = x - yi - i$

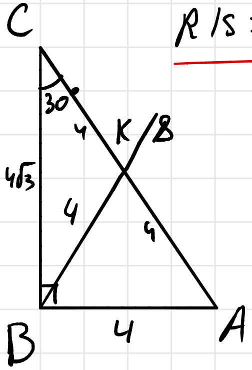
$6x - 6xi - 3i - 3 = x - yi - i$

$\begin{cases} 6x - 3 = x \\ -6x - 3 = -y - 1 \end{cases} \Leftrightarrow \begin{cases} 6x - x = 3 \\ 6x + 3 = y + 1 \end{cases} \Leftrightarrow \begin{cases} x = \frac{3}{5} \\ 6 \cdot \frac{3}{5} + 3 = y + 1 \end{cases} \Leftrightarrow \begin{cases} x = \frac{3}{5} \\ y = \frac{8}{5} \end{cases}$

R/s: $x = 3/5$; $y = 8/5$.

(7) $m(\angle B) = 90^\circ$
$m(\angle C) = 30^\circ$
$AB = 4$ cm
$[BK$ – mediană

$A_{\triangle BCK}$ – ?



În $\triangle ABC$, conform T.∢30 $\Rightarrow$
$AC = 2 \cdot 4 = 8$ cm și $CB = 4\sqrt{3}$ cm

BK - mediană $\Rightarrow$
$A_{\triangle BCK} = \frac{1}{2} A_{\triangle ABC}$

$A_{\triangle BCK} = \frac{1}{2} \cdot \frac{1}{2} \cdot 4 \cdot 4\sqrt{3} = 4\sqrt{3}\text{ cm}^2$.

(8) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x\sin x + \cos x$
pe $[0; \pi/2]$.

$f'(x) = (x\sin x + \cos x)'$

$$f'(x) = x'\sin x + x(\sin x)' - \sin x = \sin x + x\cos x - \sin x$$

$$f'(x) = x\cos x = 0 \Rightarrow \left[\begin{array}{l} x=0 \\ \cos x = 0 \end{array}\right. \xRightarrow{x\in\left[0;\frac{\pi}{2}\right]} \left[\begin{array}{l} x=0 \\ x=\frac{\pi}{2} \end{array}\right.$$

$$f(0) = 0\sin 0 + \cos 0 = 1$$

$$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = \frac{\pi}{2}$$

R/S: 1 – valoarea minimă globală

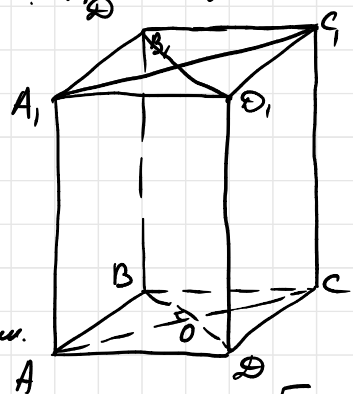
$\frac{\pi}{2}$ – valoarea maximă globală

(9) $p = m/n$ $\quad n = 9\cdot 9\cdot 8\cdot 7\cdot 6$ $\quad m = 9\cdot 8\cdot 7\cdot 6\cdot 1 + 8\cdot 8\cdot 7\cdot 6\cdot 1$

$$p = \frac{\cancel{8\cdot 7\cdot 6}(9+8)}{9\cdot 9\cdot \cancel{8\cdot 7\cdot 6}} = \frac{17}{81}.$$

(10) $A_{romb} = 18\sqrt{3}\ cm^2$ $\quad m(\angle ABC) = 60^\circ$. $\quad A_{\oslash} = 72\ cm^2$

A_d – ? $\quad A_{BDD'B'}$ – ?



$$A_{romb} = a^2\sin\alpha = 18\sqrt{3} \Leftrightarrow a^2\frac{\sqrt{3}}{2} = 18\sqrt{3}$$

$a^2 = 36 \Rightarrow a = 6$ cm. $AB = 6$ cm.

În $\triangle ABO$, $m(\angle A) = 30^\circ \Rightarrow OB = 3$ cm și

$AO = 3\sqrt{3}$ cm. Avem că $AC = 6\sqrt{3}$ cm; $BD = 6$ cm.

$$A_{AA_1C_1C} = 72\ cm^2 \Leftrightarrow H\cdot 6\sqrt{3} = 72 \Leftrightarrow H = \frac{72}{6\sqrt{3}} = \frac{72\sqrt{3}}{18} = 4\sqrt{3}\ cm$$

$$A_{BDD'B'} = H\cdot BD = 4\sqrt{3}\cdot 6 = 24\sqrt{3}\ cm^2.$$

(11) $$\log_2\left|\frac{x-1}{x+2}\right| \le 0 \Leftrightarrow \begin{cases} \left|\frac{x-1}{x+2}\right| > 0 \\ \left|\frac{x-1}{x+2}\right| \le 1 \end{cases} \Leftrightarrow \begin{cases} \frac{x-1}{x+2} \ne 0 \\ -1 \le \frac{x-1}{x+2} \le 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x \ne 1 \\ x \ne -2 \\ \frac{x-1}{x+2} \ge -1 \\ \frac{x-1}{x+2} \le 1 \end{cases} \Leftrightarrow \begin{cases} x\ne 1, x \ne -2 \\ \frac{x-1+x+2}{x+2} \ge 0 \\ \frac{x-1-x-2}{x+2} \le 0 \end{cases} \Leftrightarrow$$

$$\begin{cases} x \neq 1 \\ x \neq -2 \\ \dfrac{2x-1}{x+2} \geq 0 \\ \dfrac{-3}{x+2} \leq 0 \end{cases} \Leftrightarrow \begin{cases} x \neq 1 \\ x \neq -2 \\ x \in (-\infty; -2) \cup [\frac{1}{2}; +\infty) \\ x+2 > 0 \end{cases} \Leftrightarrow$$

$$\begin{cases} x \neq 1 \\ x \neq -2 \\ x \in (-\infty; -2) \cup [1/2; +\infty) \\ x \in (-2; +\infty) \end{cases} \Rightarrow x \in [1/2; 1) \cup (1; +\infty)$$

$$S = [1/2; 1) \cup (1; +\infty)$$

(12) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 1+2x$ $\quad F(x) - ?$ $\quad f$ - tg la G_F

Ecuația tangentei la graficul funcției F este

$y = F(x_0) + F'(x_0)(x - x_0)$; pe de altă parte $y = 2x+1$

$$F(x) = \int (1+2x)\,dx = x + x^2 + C$$

$$F'(x_0) = m = 2 \Leftrightarrow 1 + 2x_0 = 2 \Leftrightarrow x_0 = \frac{1}{2}$$

$$F(x_0) = F\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{4} + C = \frac{3}{4} + C$$

$$2x - 1 = \frac{3}{4} + C + 2\left(x - \frac{1}{2}\right) \Leftrightarrow 2x - 1 = \frac{3}{4} + C + 2x - 1$$

$\Leftrightarrow C = -\frac{3}{4}$ $\qquad$ R/s: $F: \mathbb{R} \to \mathbb{R}$, $f(x) = x + x^2 - \frac{3}{4}$

Testul 18

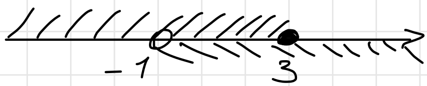
1) $\left(2^{-\frac{1}{3}}\right)^{-6} = 2^{2} = \boxed{4}$; 2) $>$; 3) 4π

4) $\log_{\frac{1}{2}} 9 + \log_2 18 = -\log_2 9 + \log_2 18 = \log_2 \frac{18}{9} = \log_2 2 = 1$

R/S: 1

5) $2 - \sqrt{3-x} > 0 \Rightarrow \sqrt{3-x} < 2 \Rightarrow 3 - x < 4 \Rightarrow x > -1$

DVA $= (-\infty; 3]$



R/S: $S = (-1; 3]$

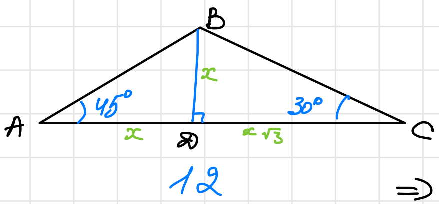
6) $z^2 - 2z = -2 \Rightarrow z^2 - 2z + 2 = 0$

$\Delta = 4 - 8 = -4$; $\sqrt{\Delta} = 2i$

$z_1 = \frac{2-2i}{2} = \frac{2}{2} - \frac{2i}{2} = 1 - i$; $z_2 = 1 + i$

R/S: $S = \{1 - i; 1 + i\}$

7)



$m(\angle C) = 30° \Rightarrow BD = x$; $DC = x\sqrt{3}$

$m(\angle A) = 45° \Rightarrow AD = BD = x$

$AC = 12 \Rightarrow x + x\sqrt{3} = 12 \Rightarrow$

$\Rightarrow x(1+\sqrt{3}) = 12 \Rightarrow x = {}^{\sqrt{3}-1)}\frac{12}{1+\sqrt{3}} = \frac{12(\sqrt{3}-1)}{3-1} \Rightarrow$

$\Rightarrow x = 6(\sqrt{3}-1)$; $\mathcal{A}_{\Delta ABC} = \frac{1}{2} \cdot BD \cdot AC = \frac{1}{2} \cdot 6(\sqrt{3}-1) \cdot 12 = 36(\sqrt{3}-1)\ (cm^2)$

R/S: $36(\sqrt{3}-1)\ cm^2$

8) $f'(x) = 2x - 1$

$f'(x) = 0$

$2x - 1 = 0$

$x = \frac{1}{2}$

x	0		$\frac{1}{2}$		1
f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

$f(0) = \frac{1}{2}$

$f(1) = \frac{1}{2}$

R/S: $f_{max}(0) = f_{max}(1) = \frac{1}{2}$

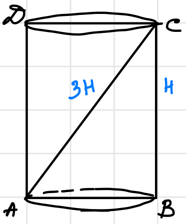
Testul 18

9) B, A, C, A, L, A, U, R, E, A, T $\longrightarrow$ $A = \{ \text{„} \angle ACAT \text{"} \}$

$$P(A) = \frac{1}{11} \cdot \frac{4}{10} \cdot \frac{1}{9} \cdot \frac{3}{8} \cdot \frac{1}{7} = \frac{1}{4620}$$

R/S: $\frac{1}{4620}$

10)



$AC = 3H$

Th. Pitagora în ΔABC: $AB^2 = 9H^2 - H^2 = 8H^2 \Rightarrow AB = 2\sqrt{2}H \Rightarrow R = H\sqrt{2}$

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R/S: 4 cm

11)

$$\int_1^9 \frac{1}{2x + 2\sqrt{x}}\,dx = \left| \begin{array}{l} \sqrt{x} = t \\ dt = \frac{1}{2\sqrt{x}}dx \\ dx = 2\sqrt{x}\,dt \\ x = 1 \Rightarrow t = 1 \\ x = 9 \Rightarrow t = 3 \end{array} \right| = \int_1^3 \frac{1}{2t^2 + 2t} \cdot 2t\,dt = \int_1^3 \frac{2t}{2t(t+1)}dt = \int_1^3 \frac{1}{t+1}dt = \ln|t+1| \Big|_1^3 =$$

$$= \ln 4 - \ln 2 = \ln 2$$

R/S: $\ln 2$

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$x_1 = \frac{2m - 1 - 1}{2} = \frac{2m - 2}{2} = m - 1$

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Înlocuim în ec. inițială

$x_1 = m \Rightarrow \sqrt{m} = \sqrt{m}$

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R/S: $m \in [-1; 0)$

Testul 18

9) $B; A; C; A; L; A; U; R; E; A; T \longrightarrow A = \{ „\angle ACAT" \}$

$$P(A) = \frac{1}{11}\cdot\frac{4}{10}\cdot\frac{1}{9}\cdot\frac{3}{8}\cdot\frac{1}{7} = \frac{1}{4620}$$

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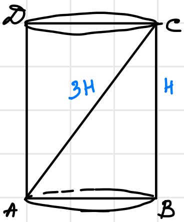
10) 

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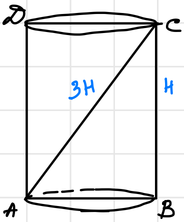
10) 

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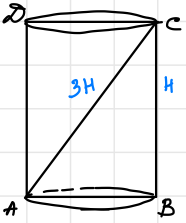

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Înlocuim în ec. inițială

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R/s: $m \in [-1; 0)$

Test 19

1) $3^{\log_9 2} = 3^{\log_{3^2} 2} = 3^{\frac{1}{2}\log_3 2} = 3^{\log_3 2^{\frac{1}{2}}} = 2^{\frac{1}{2}} = \underline{\sqrt{2}}$

2) $f(1) - f(-1) = 1^2 - (1+1) = 1 - 2 = \underline{-1}$

3) $m(\angle BAC) = \underline{30^\circ}$

4) $625^{\frac{1}{2}} \cdot \left(\frac{1}{16}\right)^{-\frac{1}{4}} = 25^{2\cdot\frac{1}{2}} \cdot 16^{\frac{1}{4}} = 25 \cdot 2^{4\cdot\frac{1}{4}} = 25 \cdot 2 = \underline{50}$

5) $1 - 25^x \cdot 5^{x+1} \geq 0 \Leftrightarrow -5^{2x} \cdot 5^{x+1} \geq -1 \Leftrightarrow$

$\Leftrightarrow 5^{2x} \cdot 5^{x+1} \leq 1 \Leftrightarrow 5^{2x+x+1} \leq 5^0 \Rightarrow$

$2x + x + 1 \leq 0 \Leftrightarrow 3x + 1 \leq 0 \Leftrightarrow 3x \leq -1 \Leftrightarrow x \leq -\frac{1}{3}$

$-\frac{1}{3}$ $\qquad S = \underline{\left(-\infty; -1/3\right]}$

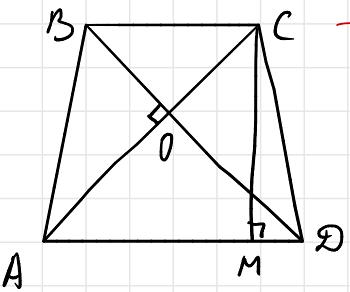
6) $2x^2 - 3x + 3 = 0 \qquad \Delta = 9 - 4\cdot 2\cdot 3 = 9 - 24 = -15$

$\sqrt{\Delta} = \sqrt{-15} = \sqrt{15}\, i$

$x_1 = \frac{3 - \sqrt{15}\, i}{4} = \frac{3}{4} - \frac{\sqrt{15}}{4} i \qquad x_2 = \frac{3}{4} + \frac{\sqrt{15}}{4} i$

R/s: $\underline{S = \left\{ \frac{3}{4} \pm \frac{\sqrt{15}}{4} i \right\}}$

7)



$BC = a, \; AD = b \qquad m = \frac{a+b}{2} = 8$

$a + b = 16 \qquad A_{trap} = m \cdot h = 8 \cdot CM$

$AM = \frac{AD + BC}{2} = \frac{b+a}{2} = 8$ cm

$AO = OD \Rightarrow \Delta AOD$ – drep. isoscel cu

$m(\angle OAD) = 45^\circ \Rightarrow \Delta ACM$ – drep. isoscel

cu $AM = CM = 8$ cm.

R/s: $\underline{A_{trap} = 8 \cdot 8 = 64 \text{ cm}^2}$

Test 19

8) $f(x)=\dfrac{x}{x^2+1}$ $\quad f'(x)=\left(\dfrac{x}{x^2+1}\right)'=\dfrac{x'(x^2+1)-x(x^2+1)'}{(x^2+1)^2}$

$f'(x)=\dfrac{1-x^2}{(x^2+1)^2}=0 \Rightarrow 1-x^2=0 \Leftrightarrow x=\pm 1.$

$x_{min}=-1 \quad y_{min}=-\frac{1}{2} \qquad x_{max}=1 \quad y_{max}=\frac{1}{2}$

R/s: $-1/2$ - valoarea minimă locală; $1/2$ - valoarea maximă locală

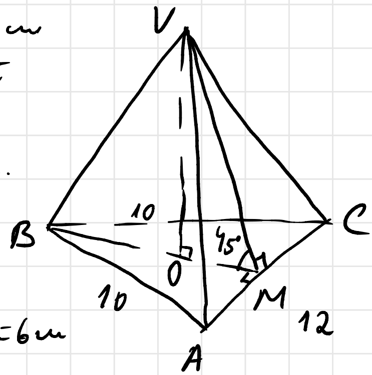
9) 10 manuale $\quad$ 1m $\quad$ 1b $\quad$ {m și b alături}

$n=10! \qquad m=2\cdot 8! \qquad p=\dfrac{2\cdot 8!}{10!}=\dfrac{1}{45}$ $\quad$ R/s: $\dfrac{1}{45}$

10) $V-?$ $\quad AB=BC=10\text{ cm} \quad AC=12\text{ cm}$

unghiuri diedre de la bază congruente de 45°

O - centrul cercului înscris $\triangle ABC$.



$A_\triangle=p r \Leftrightarrow r=\dfrac{A_\triangle}{p}$

$A_\triangle=\frac{1}{2}a h_a$. $\quad$ în $\triangle ABM$, cu $AM=6$ cm avem $BM=8$ cm.

$A_{\triangle ABC}=\dfrac{12\cdot 8}{2}=48\text{ cm}^2$

$p=\dfrac{10+10+12}{2}=16\text{ cm}$, $r=OM=\dfrac{48}{16}=3\text{ cm}.$

$\triangle VOM$ - drep. isoscel cu $VO=OM=3$ cm

$V=\frac{1}{3}A_b\cdot H=\frac{1}{3}\cdot 48\cdot 3=48\text{ cm}^3.$

11) $\dfrac{\cos x+\sin x}{\sqrt{5x-2x^2+3}}=0 \Rightarrow \begin{cases}\cos x+\sin x=0\\ 5x-2x^2+3>0\end{cases} \Leftrightarrow$

$$\Leftrightarrow \begin{cases} \sin x = -\cos x \\ 2x^2 - 5x - 3 < 0 \end{cases} \Leftrightarrow \begin{cases} tg\, x = -1 \\ x \in (-1/2;\ 3) \end{cases} \Leftrightarrow$$

$$\begin{cases} x = -\frac{\pi}{4} + \pi k,\ k \in \mathbb{Z} \\ x \in (-1/2;\ 3) \end{cases} \Rightarrow x = \frac{3\pi}{4}. \quad \underline{S = \left\{\frac{3\pi}{4}\right\}}$$

12) $|6^x + 3 - a| = 2a \quad card\, S = 0 \Leftrightarrow S = \emptyset \quad a - ?$

$a < 0 \Leftrightarrow a \in (-\infty; 0) \Rightarrow S = \emptyset$

$a = 0 \Rightarrow |6^x + 3| = 0 \Leftrightarrow 6^x + 3 = 0 \Leftrightarrow 6^x = -3 \Rightarrow S = \emptyset$

$$a > 0 \Rightarrow \left[\begin{array}{l} 6^x + 3 - a = 2a \\ 6^x + 3 - a = -2a \end{array}\right. \Leftrightarrow \left[\begin{array}{l} 6^x = 3a - 3 \\ 6^x = -a - 3 \end{array}\right. \overset{S = \emptyset}{\Rightarrow}$$

$$\begin{cases} 3a - 3 \le 0 \\ -a - 3 \le 0 \end{cases} \Leftrightarrow a \in [-3; 1] \overset{a > 0}{\Rightarrow} a \in (0; 1]$$

$\underline{R/s:\ a \in (-\infty; 1]}$

Test 20

1) $\sqrt{\log_3 81} = \sqrt{\log_3 3^4} = \sqrt{4} = 2$

2) $f\left(-\frac{b}{2a}\right) \boxed{>} 0$

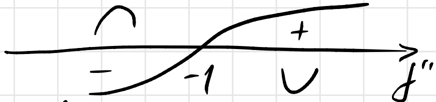
3) $m(\smile AB) = 140^\circ$

4) $4a$

Test 20

8) $f(x) = 2x^3 + 6x^2 - 1$

$f'(x) = 6x^2 + 12x$; $f''(x) = 12x + 12 = 0 \Rightarrow x = -1$



R/s: funcția f este convexă pe $[-1; +\infty)$ și concavă pe $[-\infty; -1]$.

9) $p = \frac{m}{n}$ $\quad n = 6!$ $\quad m = 2 \cdot 5!$ $\quad p = \frac{2 \cdot 5!}{6!} = \frac{2}{6} = \frac{1}{3}$

R/s: $\frac{1}{3}$

10) $R = 2\sqrt{6}$ cm $\quad AB = 5\sqrt{3}$ cm $\quad m(\angle AVB) = 120°$

$V - ?$ $\quad V = \frac{1}{3} A_B \cdot H = \frac{1}{3} \pi R^2 H$

$A_B = \pi R^2 = \pi (2\sqrt{6})^2 = 24\pi$

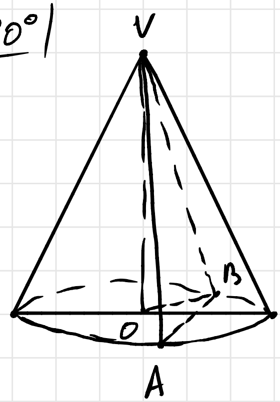
conform t. cosinusului în $\triangle VAB$

$(5\sqrt{3})^2 = G^2 + G^2 - 2G \cdot G \cdot \cos 120°$

$G = 5$ cm $\quad VA = VB = 5$ cm.

CTP în $\triangle VOB$. $\quad VO = 1$ cm

$V = \frac{1}{3} \cdot 24\pi \cdot 1 = 8\pi$ cm^3



R/s: 8π cm^3

11) $\displaystyle\int_{\pi/2}^{\pi} \frac{\cos x}{1 + 2\sin x}\,dx = \left| \begin{array}{l} 1 + 2\sin x = t \\ 2\cos x\,dx = dt \\ x = \pi/2,\ t = 3 \\ x = \pi,\ t = 1 \end{array} \right| = -\frac{1}{2}\int_1^3 \frac{dt}{t} = -\frac{1}{2} \ln|t| \Big|_1^3 =$

$= -\frac{1}{2}(\ln 3 - \ln 1) = -\frac{\ln 3}{2}$.

12) $\Delta = 0$ - sistemul are o infinitate de soluții

$\Delta = \begin{vmatrix} 1 & -1 & a \\ -1 & 1 & a-2 \\ a & 0 & -1 \end{vmatrix} = -4a^2 + 4a - 2 = 0$.

$\Delta = -4 \quad a \in \emptyset$ $\quad$ R/s: $a \in \emptyset$.

Test 21

1) $\boxed{-3} < \sqrt[3]{-9} < \boxed{-2}$

2) $f: R \to R$, $f(x) = x^2 + |x|$

$f(-x) = (-x)^2 + |-x| = x^2 + |x|$ – $\boxed{\text{pară}}$

3) $m(\angle BCD) = 150^\circ$

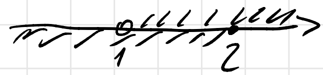
4) $\left(\frac{1}{\sqrt{2}}\right)^{-2} + 2\log_2 \sqrt{8} = \sqrt{2}^2 + \log_2 \sqrt{8}^2 = 2 + \log_2 8 =$

$= 2 + \log_2 2^3 = 2 + 3 = 5.$

5) $\frac{1}{\sqrt{x-1}} \geq 1$

1) DVA: $x - 1 > 0 \Leftrightarrow x > 1$

2) $1 \geq \sqrt{x-1} \Leftrightarrow \sqrt{x-1} \leq 1 \Leftrightarrow x - 1 \leq 1 \Leftrightarrow x \leq 2$

3)


$S = (1; 2]$

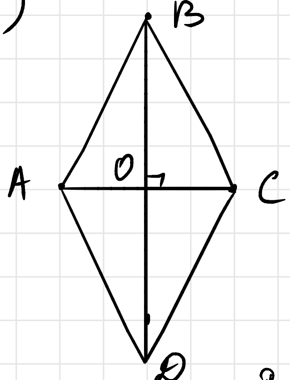
6) $z = a + bi$, $a, b \in R$

$\bar{z} = \frac{z-3}{1-i} \Leftrightarrow \bar{z}(1-i) = z - 3 \Leftrightarrow$

$(a - bi)(1 - i) = a + bi - 3 \Leftrightarrow a - ai - bi - b = a + bi - 3$

$a - b = a - 3$ $\quad$ $-a - b = b$ $\quad$ $z = -6 + 3i$

$b = 3$ $\quad$ $a = -6$

7)


$\frac{BD}{AC} = \frac{4}{3}$ $\quad$ $BC = 20$ cm $\quad | \quad A_{ABCD}$ – ?

$A_{tr.} = \frac{BD \cdot AC}{2}$

$BD = 4K \Rightarrow BO = 2K$

$AC = 3K \Rightarrow OC = \frac{3K}{2}$

În ΔBOC, CTP: $20^2 = 4K^2 + \frac{9K^2}{4} \Rightarrow K = 8$

$BD = 32$ cm $\quad AC = 24$ cm $\quad A_{trp.} = 384$ cm^2

Test 21

8) $f:[-1;2]\to\mathbb{R},\quad f(x)=-x^2+x+2$

$$A(\Gamma_f)=\int_{-1}^{2}(-x^2+x+2)\,dx=\left(\frac{-x^3}{3}+\frac{x^2}{2}+2x\right)\Big|_{-1}^{2}=$$

$$=\left(-\frac{8}{3}+2+4\right)-\left(\frac{1}{3}+\frac{1}{2}-2\right)=-3+8-\frac{1}{2}=5-\frac{1}{2}=4{,}5$$

R/s: $\frac{9}{2}$.

9) $p=m/n \qquad n=6! \qquad m=6\cdot 4! \qquad p=\frac{6\cdot 4!}{6!}=\frac{1}{5}$ R/s: $1/5$

10) $m(\angle C_1MC)=45^\circ$

$CC_1=B_1B=AA_1=4\sqrt{3}$ cm $=CM$.

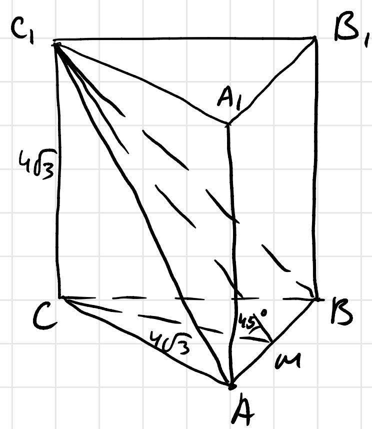
în $\triangle ABC$, notăm $AC=2a \Rightarrow AM=a$

CTP. în $\triangle AMC$, $(2a)^2=a^2+(4\sqrt{3})^2\Rightarrow a=4$

$AC=AB=CB=8$ cm.

$$V=\frac{1}{3}A_b\cdot H=\frac{1}{3}\frac{a\,h_a}{2}H=\frac{1}{3}\cdot\frac{8\cdot 4\sqrt{3}\cdot 4\sqrt{3}}{2}$$

$V=64\text{ cm}^3$.



11) $\det\neq 0$ – matricea este inversabilă

$\text{ctg}\,x\cdot 2\sin x-1\neq 0 \Leftrightarrow 2\frac{\cos x}{\sin x}\cdot\sin x-1\neq 0\Rightarrow$

$\begin{cases}\sin x\neq 0\\ \cos x\neq\frac{1}{2}\end{cases}\Rightarrow\begin{cases}x\neq\pm\frac{\pi}{3}+2\pi k,\ k\in\mathbb{Z}\\ x\neq\pi n,\ n\in\mathbb{Z}\end{cases}$

R/s: $x\in\mathbb{R}\setminus\left\{\pm\frac{\pi}{3}+2\pi k, k\in\mathbb{Z}\right\}\cup\{\pi n, n\in\mathbb{Z}\}$

12) $f(x)=\sin 3x-(x+3)(a^2-2a)$; $f\nearrow$ pe $\mathbb{R}\Rightarrow f'(x)\geq 0,\ \forall x\in\mathbb{R}$

$f'(x)=3\cos 3x-a^2+2a\geq 0,\ \forall x\in\mathbb{R}$

$\cos 3x\geq (a^2-2a)/3,\ \forall x\in\mathbb{R}$

Deoarece $\cos 3x\in[-1;1]\Rightarrow\frac{a^2-2a}{3}\leq 1\Leftrightarrow a^2-2a-3\leq 0\Leftrightarrow$

$a\in[-1;3]$ R/s: $a\in[-1;3]$.

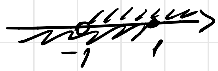
Test 22

1. $-\log_3 \frac{1}{2} \boxed{>} 0 \qquad -\log_3 \frac{1}{2} = -\log_3 2^{-1} = -(-1)\log_3 2 = \log_3 2.$

2. $(-\infty; 1)$ 3. $18\pi.$

4. $8^{\frac{5}{3}} - \sqrt[3]{27} + \left(\frac{1}{3}\right)^{-1} = 2^{3\cdot\frac{5}{3}} - \sqrt[3]{3^3} + 3^1 = 2^5 - 3 + 3 = 32.$

5. $\log_4 (x+1) \le \frac{1}{2}$

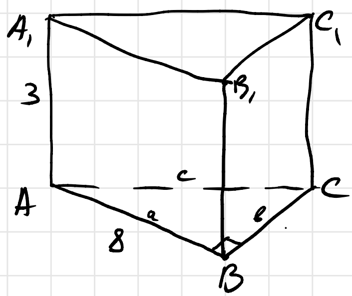
1. DVA: $x+1 > 0 \Leftrightarrow x > -1$
2. $x+1 \le 4^{\frac{1}{2}} \Leftrightarrow x+1 \le 2 \Leftrightarrow x \le 1$
3)  $S = (-1; 1]$

6. $$z = \begin{vmatrix} 2 & i & -i \\ x & -1 & -1 \\ x & i & i \end{vmatrix} = -2i - xi^2 - xi - xi + 2i - xi^2 =$$
$$= 2x - xi$$

$$2x - xi = 2 - 2i \Rightarrow \begin{cases} 2x = 2 \\ -2x = -2 \end{cases} \Rightarrow x = 1 \qquad R/S: x = 1.$$

7. $AB = 8\,cm;\ r = 3\,cm,\ r = AA_1$

$V_{prismei} - ?$ $\quad V = A_b \cdot H$



$$r = \frac{a+b-c}{2} \Leftrightarrow \begin{aligned} 8+b-c &= 6 \\ b-c &= -2 \\ c-b &= 2 \end{aligned}$$

CTP $\quad a^2 + b^2 = c^2 \Leftrightarrow 8^2 + b^2 = c^2$

$$c^2 - b^2 = 64 \Leftrightarrow (c-b)(c+b) = 64 \Leftrightarrow 2\cdot(c+b) = 64 \Leftrightarrow c+b = 32$$

$$\begin{cases} c+b = 32 \\ c-b = 2 \end{cases} \Leftrightarrow \begin{cases} c = 17 \\ b = 15 \end{cases} \qquad A_b = \frac{ab}{2} = \frac{8\cdot 15}{2} = 60\,cm^2$$

$$V = 60 \cdot 3 = 180\,cm^3.$$

8. $$\int_0^5 \frac{dx}{\sqrt{3x+1}} = \int_0^5 (3x+1)^{-\frac{1}{2}} dx = \left| \begin{aligned} 3x+1 &= t \\ 3dx &= dt \\ x = 0,\ t &= 1 \\ x = 5,\ t &= 16 \end{aligned} \right| =$$

$$= \frac{1}{3}\int_1^{16} t^{-\frac{1}{2}} dt = \frac{1}{3} \left.\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right|_1^{16} = \frac{2}{3}\sqrt{t}\Big|_1^{16} = \frac{2}{3}\left(\sqrt{16} - \sqrt{1}\right) = \frac{2}{3}\cdot 3 = 2.$$

R/S: 2

9) $P(A) = 2/7 \quad P(\bar{A}) = 5/7 \quad B - \{$exact 2 elevi acceptați$\}$

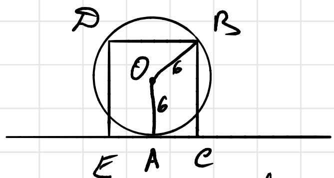
$$B = (A \cap A \cap \bar{A}) \cup (A \cap \bar{A} \cap A) \cup (\bar{A} \cap A \cap A)$$

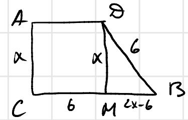
$$P = P(A) \cdot P(A) \cdot P(\bar{A}) \cdot 3 = \frac{2}{7} \cdot \frac{2}{7} \cdot \frac{5}{7} \cdot 3 = \frac{60}{343} \qquad R/s: \frac{60}{343}$$

(10) $R = 6$ | $DC - ?$

Notăm $AC = x$, $BC = 2x$

$$DM^2 + MB^2 = DB^2$$
$$x^2 + (2x-6)^2 = 6^2$$
$$x = 24/5 \quad BC = \frac{48}{5} \text{ cm}$$





R/s: $DC = \frac{48\sqrt{2}}{5}$ cm.

(11) $\frac{f'(x)}{6x+6} \le 0$; $f(x) = 2x^3 - 3x^2 - 12x$; $f'(x) = 6x^2 - 6x - 12$

$$\frac{6(x^2 - x - 2)}{6(x+1)} \le 0 \iff \frac{x^2 - x - 2}{x+1} \le 0 \quad DVA: x \ne -1$$

$$x^2 - x - 2 = 0 \iff \left[\begin{array}{l} x = -1 \\ x = 2 \end{array}\right.$$

$$S = (-\infty; -1) \cup (-1; 2]$$

(12) $\log_{a^2}(4 + x^2) \ge 1$, $\forall x \in R$.

$a \ne 0$, $a \ne \pm 1$.

Caz I p/u $0 < a^2 < 1 \Rightarrow 4 + x^2 \le a^2 \iff x^2 \le a^2 - 4$, $\forall x \in R$ - Fals

Caz II p/u $a^2 > 1 \Rightarrow 4 + x^2 \ge a^2 \iff x^2 \le a^2 - 4$, $\forall x \in R \Rightarrow$

$a^2 - 4 \le 0 \Rightarrow a \in [-2; 2]$

$$\begin{cases} a^2 > 1 \\ a \in [-2; 2] \end{cases} \iff \begin{cases} a \in (-\infty; -1) \cup (1; +\infty) \\ a \in [-2; 2] \end{cases} \Rightarrow$$

$$a \in [-2; -1) \cup (1; 2].$$

Test 23.

① $\log_{\sqrt{3}} 3 \ \boxed{=} \ \log_{\frac{1}{2}} \frac{1}{4}$ $\log_{\sqrt{3}} 3 = 2$; $\log_{\frac{1}{2}} \frac{1}{4} = 2$.

② $(-1;0]$ ③ $C'D = \sqrt{2}$ cm

④ $\sqrt[3]{-8} + 25^{\log_5 3} = -2 + 5^{2\log_5 3} = -2 + 5^{\log_5 9} = -2+9 = 7$

⑤

$$\det A = \begin{vmatrix} 2+i & 1 & -1 \\ i & 1 & 0 \\ 1+bi & 2 & 2-i \end{vmatrix} = (2+i)(2-i) - 2i + (1+bi) - i(2-i) =$$
$$= 5 + bi - 4i$$

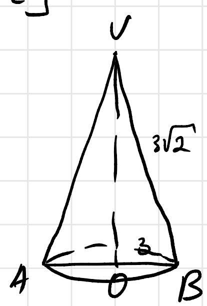
$\det A \in \mathbb{R} \Leftrightarrow 5 + bi - 4i \in \mathbb{R} \Rightarrow b - 4 = 0 \Leftrightarrow b = 4.$

R/s: $b = 4$.

⑥ $4^x - 2^{x+2} \le 0 \Leftrightarrow 2^{2x} \le 2^{x+2} \Leftrightarrow 2x \le x+2 \Leftrightarrow$
$2x - x \le 2 \Leftrightarrow x \le 2 \Rightarrow S = (-\infty; 2]$

⑦ $A_{lat} = 9\pi\sqrt{2}$ cm^2
$A_{tot} = \frac{9}{\sqrt{2}-1}\pi$ cm^2
$m(\angle BVO) - ?$

$A_{lat} = \pi R G$
$A_{tot} = \pi R G + \pi R^2$



$\pi R G = 9\pi\sqrt{2}$
$RG = 9\sqrt{2}$

$\pi R(G+R) = \frac{9}{\sqrt{2}-1}\pi$

$RG + R^2 = \frac{9}{\sqrt{2}-1} \Leftrightarrow 9\sqrt{2} + R^2 = \frac{9(\sqrt{2}+1)}{2-1} \Leftrightarrow R^2 = 9 \Rightarrow R = 3$ cm

$G = \frac{9\sqrt{2}}{R} = 3\sqrt{2}$ cm $\Rightarrow VO = 3$ cm ΔVOB - drept isoscel

R/s: $m(\angle OVB) = 45°$.

⑧ $f : [0; \frac{\pi}{2}] \to \mathbb{R}$, $f(x) = -x\sin x - \cos x$

$f'(x) = (-\cos x)' - (x\sin x)' = \sin x - x'\sin x + x(\sin x)'$

$f'(x) = x\cos x = 0 \Rightarrow \left[\begin{array}{l} x = 0 \\ \cos x = 0 \end{array}\right. \xRightarrow{x \in [0;\frac{\pi}{2}]} \left[\begin{array}{l} x = 0 \\ x = \frac{\pi}{2} \end{array}\right.$

$f(0) = -0\sin 0 - \cos 0 = -1$ $f(\frac{\pi}{2}) = -\frac{\pi}{2}\sin\frac{\pi}{2} - \cos\frac{\pi}{2} = -\frac{\pi}{2}$

R/S: $-\frac{\pi}{2}$ - val. minimă globală ; -1 - valoarea maximă globală

9) A – {cel puțin 1 bilet câștig} $\bar{A}$ – {nici un bilet câștig}

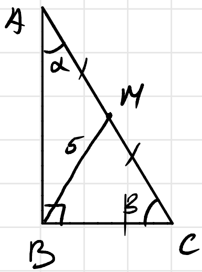
$P(\bar{A}) = \frac{m}{n}$; $n = C_{40}^{4} = 37 \cdot 38 \cdot 13 \cdot 5$

$m = C_{20}^{4} = 17 \cdot 15 \cdot 19$ $P(\bar{A}) = \frac{51}{962}$

$P(A) = 1 - \frac{51}{962} = \frac{911}{962}$ R/S: $\frac{911}{962}$.

10) BM – mediană
$BM = 5$ cm
$\alpha/\beta = 1/2$

$A_{\triangle ABC}$ – ?



$AC = 2BM = 10$ cm

$\begin{cases} \alpha + \beta = 90^\circ \\ \beta = 2\alpha \end{cases} \Rightarrow \begin{cases} \alpha = 30^\circ \\ \beta = 60^\circ \end{cases}$

$BC = \frac{1}{2} AC = 5$ cm

$AB = 5\sqrt{3}$ cm

$A_{\triangle ABC} = \frac{5\sqrt{3} \cdot 5}{2} = \frac{25\sqrt{3}}{2}$ cm^2

11) $\int_1^9 \frac{dx}{x + \sqrt{x}} = \left| \begin{array}{l} \sqrt{x} = t \\ x = t^2 \\ dx = 2t\,dt \\ x = 1,\ t = 1 \\ x = 9,\ t = 3 \end{array} \right| = \int_1^3 \frac{2t\,dt}{t^2 + t} = 2\int_1^3 \frac{dt}{t+1} =$

$= 2\int_1^3 \frac{d(t+1)}{t+1} = 2\ln(t+1)\Big|_1^3 = 2(\ln 4 - \ln 2) = 2\ln 2 = \ln 4.$

12) $|2^x - a + 2| = a$ card $S = 0 \Leftrightarrow S = \emptyset$

1) $a < 0 \Rightarrow S = \emptyset$
2) $a = 0 \Rightarrow |2^x + 2| = 0 \Leftrightarrow 2^x + 2 = 0 \Rightarrow S = \emptyset$
3) $a > 0 \Rightarrow \left[\begin{array}{l} 2^x - a + 2 = a \\ 2^x - a + 2 = -a \end{array} \right. \Leftrightarrow \left[\begin{array}{l} 2^x = 2a - 2 \\ 2^x = -2 \end{array} \right. \Rightarrow 2^x = 2a - 2$

$S = \emptyset \Rightarrow 2a - 2 \leq 0 \Leftrightarrow a \leq 1$ R/S: $a \in (-\infty; 1]$.

Test 24

① $\log_8 2 = \log_{2^3} 2^1 = \boxed{\frac{1}{3}}$ ② nici pară, nici impară

③ $m(\angle ADB) = 45^\circ$

④ $\left(\frac{9}{4}\right)^{\frac{1}{2}} - \left(\frac{2}{3}\right)^{-1} + \sqrt[3]{125} = \left(\frac{3}{2}\right)^{2\cdot\frac{1}{2}} - \left(\frac{3}{2}\right)^{1} + \sqrt[3]{5^3} =$

$= \frac{3}{2} - \frac{3}{2} + 5 = 5.$

⑤ $z = 2(i^4 + 3i^3)^2.$ $i^4 = i^2 \cdot i^2 = -1\cdot(-1) = 1$

$i^3 = i^2 \cdot i = -i$

$z = 2(1-3i)^2 = 2(1-3i)(1-3i) = 2(1-3i-3i+9i^2)$

$z = 2(1-6i-9) = 2(-8-6i) = -16-12i$

$|z| = \sqrt{16^2+12^2} = \sqrt{256+144} = \sqrt{400} = 20.$ R/S: 20

⑥ $(12x - x^2)\log_2 x = 0$

1) DVA: $x > 0$ 2) $12x - x^2 = 0$ sau $\log_2 x = 0$

$-x^2 + 12x = 0$ $x = 2^0$

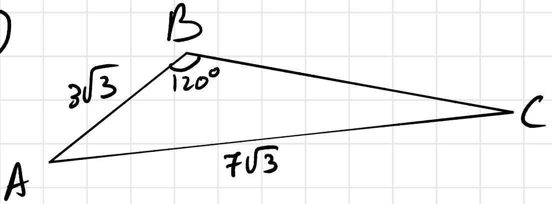
$x^2 - 12x = 0$ $x = 1 \in DVA$

$x(x-12) = 0$

$\left[\begin{array}{l} x = 0 \\ x - 12 = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = 0 \notin DVA \\ x = 12 \in DVA \end{array}\right.$

$S = \{12; 1\}$

⑦



1) $\frac{AC}{\sin 120^\circ} = 2R \Rightarrow$

$AC = 2R\sin 120^\circ = 7\sqrt{3}$ cm

2) $AC^2 = AB^2 + BC^2 - 2AB\cdot BC\cdot\cos 120^\circ$

$$BC^2 + 3\sqrt{3}\,BC - 120 = 0 \Rightarrow BC = 5\sqrt{3}\ cm.$$

$$A_\Delta = p\,r \Rightarrow r = \frac{A}{p} \quad ; \quad p = \frac{15\sqrt{3}}{2}\ cm$$

$$A_\Delta = \frac{1}{2}ab\sin\alpha = \frac{1}{2}\,5\sqrt{3}\cdot 3\sqrt{3}\cdot\frac{\sqrt{3}}{2} = \frac{45\sqrt{3}}{4}\ cm^2$$

$$r = \frac{45\sqrt{3}}{4} : \frac{15\sqrt{3}}{2} = \frac{3}{2}\ cm \qquad R: \underline{\frac{3}{2}\ cm.}$$

(8) $f:[0;4] \to \mathbb{R},\ f(x) = \dfrac{x-1}{\sqrt{x}+1}$. $A(\Gamma_f) - ?$

1) $f(x) = 0 \Rightarrow \dfrac{x-1}{\sqrt{x}+1} = 0 \Rightarrow x = 1$

2) $$A(\Gamma_f) = -\int_0^1 \frac{x-1}{\sqrt{x}+1}\,dx + \int_1^4 \frac{x-1}{\sqrt{x}+1}\,dx =$$

$$= -\int_0^1 \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}+1}\,dx + \int_1^4 \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}+1}\,dx =$$

$$= \int_0^1 (1-\sqrt{x})\,dx + \int_1^4 (\sqrt{x}-1)\,dx = \left(x - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right)\Bigg|_0^1 +$$

$$+ \left(\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - x\right)\Bigg|_1^4 = \left(x - \frac{2x^{\frac{3}{2}}}{3}\right)\Bigg|_0^1 + \left(\frac{2x^{\frac{3}{2}}}{3} - x\right)\Bigg|_1^4 =$$

$$= \left(1 - \frac{2}{3}\right) - (0-0) + \left(\frac{2\cdot 4^{\frac{3}{2}}}{3} - 4\right) - \left(\frac{2}{3} - 1\right) =$$

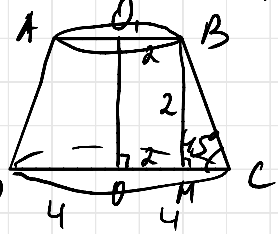
$$\frac{1}{3} + \frac{16}{3} - 4 - \frac{2}{3} + 1 = 5 - 4 + 1 = 2.$$

R/S: $\underline{2.}$

9) $p=\frac{m}{n}$; $m = 5!$ $\quad n = 6!$ $\quad p = \frac{5!}{6!} = 1/6.$

10) $R = 4$ cm
$r = 2$ cm
$m(\angle BCO) = 45^\circ$

$V - ?$



$BM = MC = 2$ cm

$$V = \frac{\pi h}{3}\left(R^2 + Rr + r^2\right)$$

$$V = \frac{\pi 2}{3}\left(16 + 4\cdot 2 + 2^2\right) = \frac{\pi\cdot 2\cdot 28}{3} = \frac{56\pi}{3}\text{ cm}^3.$$

11) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 5x + \sin 2x - 4\sqrt{3}\sin x$
$f'(x) = 0$

$$f'(x) = 5 + 2\cos 2x - 4\sqrt{3}\cos x = 0$$
$$5 + 2(2\cos^2 x - 1) - 4\sqrt{3}\cos x = 0$$
$$4\cos^2 x - 4\sqrt{3}\cos x + 3 = 0$$
$$\cos x = \frac{\sqrt{3}}{2} \Leftrightarrow$$
$$x = \pm\frac{\pi}{6} + 2\pi k, k \in \mathbb{Z}.$$

R/s: $x = \pm\frac{\pi}{6} + 2\pi k, k \in \mathbb{Z}$

12) $$\begin{cases} 3(a-5x) < 1+x \\ 2 - \frac{x}{2} > 3 + 5(x-a) \end{cases} \Leftrightarrow \begin{cases} 3a - 15x < 1 + x \\ 4 - x > 6 + 10x - 10a \end{cases}$$

$$\Leftrightarrow \begin{cases} 16x > 3a - 1 \\ 11x < 10a - 2 \end{cases} \Leftrightarrow \begin{cases} x > \frac{3a-1}{16} \\ x < \frac{10a-2}{11} \end{cases} \quad S \neq \emptyset \Rightarrow$$

$$\frac{3a-1}{16} < \frac{10a-2}{11} \Rightarrow a > \frac{21}{127}$$

R/s: $a \in \left(\frac{21}{127}; +\infty\right)$

Test 25

① $\boxed{-3} < \sqrt[3]{-24} < \boxed{-2}$ ② $E(f) = [-8; 1]$.

③ $P_{ABCD} = \boxed{7}$ cm

④ $-\log_{1/3} 3 + \left(\frac{1}{\sqrt{2}}\right)^{-2} = -1\log_{3^{-1}} 3 + \left(\frac{\sqrt{2}}{1}\right)^2 =$

$= \log_{3^{-1}} 3^{-1} + \sqrt{2}^2 = \log_3 3 + 2 = 1+2=3 \in \mathbb{N}.$

⑤ $(x^2-4x)\sqrt{7x-21} = 0$

DVA: $7x-21 \geq 0 \Leftrightarrow 7x \geq 21 \Leftrightarrow x \geq 3$

$x^2-4x=0$ $\sqrt{7x-21}=0$

$x(x-4)=0$ $7x-21=0$

$x=0 \notin$ DVA $x=3 \in$ DVA

$x=4 \in$ DVA

$S = \{3; 4\}$

⑥ $|z|$-?

$$z = \det A = \begin{vmatrix} 1 & i & 2 \\ -1 & 0 & 1 \\ 2i & -1 & 1 \end{vmatrix} = 0+2+2i^2-(0-1-i) =$$

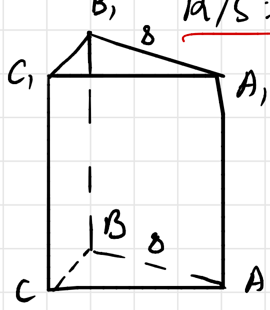
$$= 2-2+1+i = 1+i$$

$a=1,\ b=1$ $|z| = \sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2}$

R/S: $\sqrt{2}$

⑦ $AB = 8$ cm
$R = 5$ cm
$H = 2$

A_{tot} - ?



$R = \frac{AC}{2} \Rightarrow AC = 10$ cm. Conform T. Pitagora $CB = 6$cm

$A_{bazei} = \frac{6 \cdot 8}{2} = 24 \text{ cm}^2$ $\quad r = \frac{a+b-c}{2} = 2$ cm.

$H = 2$ cm. $\quad A_{lat} = P_b \cdot H = 24 \cdot 2 = 48 \text{ cm}^2$

R/S: $A_{tot} = 2 \cdot 24 + 48 = 96 \text{ cm}^2$.

8) $y - ?$ $f: \mathbb{R} \to \mathbb{R}$, $f(x) = -\frac{x^3}{3} + \frac{x^2}{2} + 2x + 1$

cea mai mare pantă $\Rightarrow f'(x_0) \to \max$.

$y = f(x_0) + f'(x_0)(x - x_0)$

$f'(x) = -x^2 + 2x + 2$

$f'(x_0) = -x_0^2 + 2x_0 + 2 \to \max$

$f'(x_0)$ este o funcție de gradul II cu $a < 0 \Rightarrow$

ea își atinge maximumul în $x_0 = -\frac{b}{2a} = 1$.

$x_0 = 1 \Rightarrow f(x_0) = f(1) = \frac{19}{6}$ $\quad f'(x_0) = f'(1) = 3$

$y = \frac{19}{6} + 3(x - 1) \Leftrightarrow y = 3x + \frac{1}{6}$

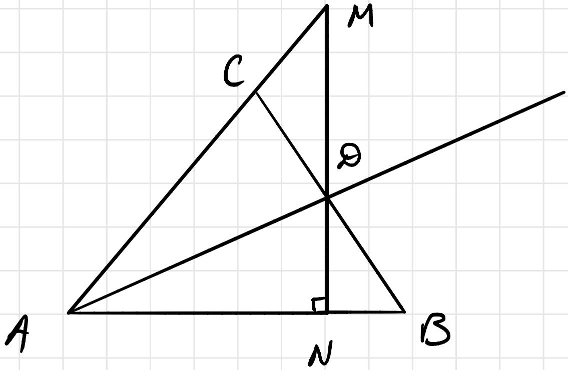
R/S: $y = 3x + \frac{1}{6}$

9) $P(A) = \frac{1}{10}$ $\quad P(\bar{A}) = \frac{9}{10}$

$P = P(A) \cdot P(A) \cdot P(\bar{A}) \cdot 3 = \left(\frac{1}{10}\right)^2 \cdot \frac{9}{10} \cdot 3 = 0,027$.

R/S: 0,027

(10)



în $\triangle ADN$, $m(\angle A) = 30^\circ \Rightarrow$
$DN = \frac{1}{2}$ cm $\quad AD = 1$ cm și
$AN = \sqrt{3}$ cm.
în $\triangle NDB$, $m(\angle B) = 45^\circ \Rightarrow$
$NB = 1$ cm, $DB = \sqrt{2}$ cm.
în $\triangle AMN$, $m(\angle A) = 60^\circ$,
$m(\angle M) = 30^\circ \Rightarrow$

$AM = 2AN = 2\sqrt{3}$ cm și
$NM = \sqrt{3} \cdot \sqrt{3} = 3$ cm
R/S: 3 cm.

(11) $\cos x \geq 0$

$$1 + 5\sin x + 2\cos^2 x = 0$$
$$1 + 5\sin x + 2(1 - \sin^2 x) = 0$$
$$2\sin^2 x - 5\sin x - 3 = 0$$

$$\left[\begin{array}{l} \sin x = -\frac{1}{2} \\ \sin x = 3 \end{array}\right. \xRightarrow{-1 \leq \sin x \leq 1} \sin x = -\frac{1}{2}$$

$$\left[\begin{array}{l} x = -\frac{\pi}{6} + 2\pi k, k \in \mathbb{Z} \\ x = -\frac{5\pi}{6} + 2\pi k, k \in \mathbb{Z} \end{array}\right. \xRightarrow{\cos x \geq 0} x = -\frac{\pi}{6} + 2\pi k, k \in \mathbb{Z}$$

R/S: $S = \left\{-\frac{\pi}{6} + 2\pi k, k \in \mathbb{Z}\right\}$

(12) $y = 2x^2$, $y = a$, $A = \frac{20\sqrt{2}}{3}$

$2x^2 = a \Rightarrow x = \pm\frac{\sqrt{2a}}{2}$

$$A = \int_{-\sqrt{2a}/2}^{\sqrt{2a}/2} (a - 2x^2)\,dx = \left(ax - \frac{2x^3}{3}\right)\Bigg|_{-\frac{\sqrt{2a}}{2}}^{\frac{\sqrt{2a}}{2}} = \frac{2a\sqrt{2a}}{3}$$

$\frac{2a\sqrt{2a}}{3} = \frac{20\sqrt{2}}{3} \Rightarrow a = \sqrt[3]{100}$ $\quad$ R/S: $\sqrt[3]{100}$.