Name_____

Complex Numbers and Polar Form

Date_____ Period____
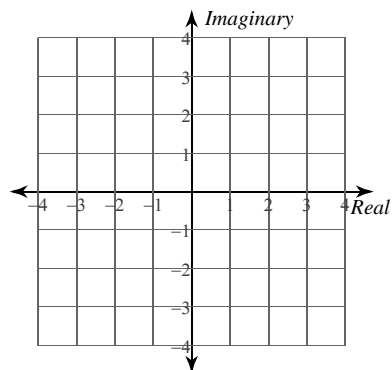
Find the absolute value.

1) $4 + 3i$

2) $-\sqrt{15} + i\sqrt{15}$

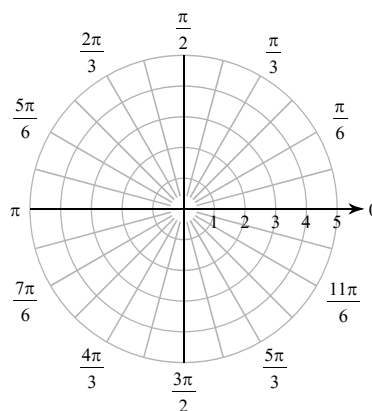3) $3\left(\cos\dfrac{3\pi}{2} + i\sin\dfrac{3\pi}{2}\right)$

4) $\sqrt{21}\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

Plot each point in the complex plane. Use rectangular coordinates when the number is given in rectangular form and polar coordinates when polar form is used.

5) $1 + 3i$

6) $3\left(\cos\dfrac{7\pi}{6} + i\sin\dfrac{7\pi}{6}\right)$

Convert numbers in rectangular form to polar form and polar form to rectangular form.

7) $-2\sqrt{3} - 2i$

8) $\sqrt{21} + i\sqrt{7}$

9) $-\dfrac{5\sqrt{2}}{2} - \dfrac{5\sqrt{2}}{2}i$

10) $2i$

11) $\sqrt{6}\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

12) $2\left(\cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}\right)$

13) $\sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

14) $\sqrt{30}\left(\cos\pi + i\sin\pi\right)$

Simplify. Write your answer in rectangular form when rectangular form is given and in polar form when polar form is given.

15) $(4 + 4i)(5 - 3i)$

16) $4\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) \cdot 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

17) $\dfrac{6 - 2i}{2 + 4i}$

18) $\dfrac{2\sqrt{6}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)}{6\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)}$

19) $(-1 - 6i)^3$

20) $\left(2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)\right)^3$

Find all *n*th roots. Write your answers in rectangular form when rectangular form is given and in polar form when polar form is given.

21) $2i, \ n = 3$

22) $6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right), \ n = 4$

Critical thinking questions:

23) Show that $-i\sqrt[23]{n}$ is a 23rd root of *ni*.

24) Solve for $x : (2 + i)x = 1 + 2i$
Hint: x is a complex number.

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Find the absolute value.

1) $4 + 3i$

   5

2) $-\sqrt{15} + i\sqrt{15}$
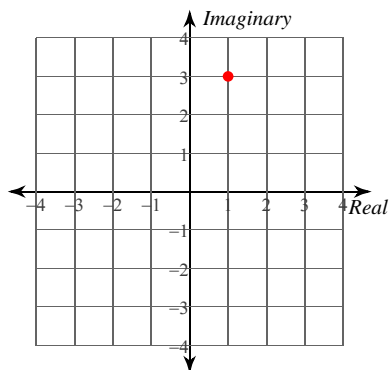
   $\sqrt{30}$

3) $3\left(\cos\dfrac{3\pi}{2} + i\sin\dfrac{3\pi}{2}\right)$
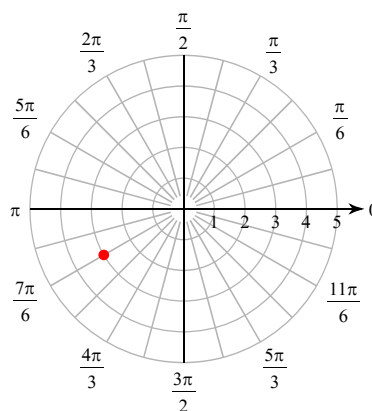
   3

4) $\sqrt{21}\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

   $\sqrt{21}$

Plot each point in the complex plane. Use rectangular coordinates when the number is given in rectangular form and polar coordinates when polar form is used.

5) $1 + 3i$

6) $3\left(\cos\dfrac{7\pi}{6} + i\sin\dfrac{7\pi}{6}\right)$

Convert numbers in rectangular form to polar form and polar form to rectangular form.

7) $-2\sqrt{3} - 2i$

   $4\left(\cos\dfrac{7\pi}{6} + i\sin\dfrac{7\pi}{6}\right)$

8) $\sqrt{21} + i\sqrt{7}$

   $2\sqrt{7}\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

9) $-\dfrac{5\sqrt{2}}{2} - \dfrac{5\sqrt{2}}{2}i$

   $5\left(\cos\dfrac{5\pi}{4} + i\sin\dfrac{5\pi}{4}\right)$

10) $2i$

   $2\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

11) $\sqrt{6}\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

   $i\sqrt{6}$

12) $2\left(\cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}\right)$

   $-1 - i\sqrt{3}$

13) $\sqrt{3}\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

$\dfrac{3}{2} + \dfrac{\sqrt{3}}{2}i$

14) $\sqrt{30}(\cos\pi + i\sin\pi)$

$-\sqrt{30}$

Simplify. Write your answer in rectangular form when rectangular form is given and in polar form when polar form is given.

15) $(4 + 4i)(5 - 3i)$

$32 + 8i$

16) $4\sqrt{2}\left(\cos\dfrac{7\pi}{4} + i\sin\dfrac{7\pi}{4}\right) \cdot 2\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

$8\sqrt{2}\left(\cos\dfrac{23\pi}{12} + i\sin\dfrac{23\pi}{12}\right)$

17) $\dfrac{6 - 2i}{2 + 4i}$

$\dfrac{1}{5} - \dfrac{7}{5}i$

18) $\dfrac{2\sqrt{6}\left(\cos\dfrac{7\pi}{6} + i\sin\dfrac{7\pi}{6}\right)}{6\left(\cos\dfrac{11\pi}{6} + i\sin\dfrac{11\pi}{6}\right)}$

$\dfrac{\sqrt{6}}{3}\left(\cos-\dfrac{2\pi}{3} + i\sin-\dfrac{2\pi}{3}\right)$

19) $(-1 - 6i)^3$

$107 + 198i$

20) $\left(2\left(\cos\dfrac{7\pi}{6} + i\sin\dfrac{7\pi}{6}\right)\right)^3$

$8\left(\cos\dfrac{7\pi}{2} + i\sin\dfrac{7\pi}{2}\right)$

Find all _nth_ roots. Write your answers in rectangular form when rectangular form is given and in polar form when polar form is given.

21) $2i$, $n = 3$

$1.09 + 0.63i$
$-1.09 + 0.63i$
$-1.26i$

22) $6\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right)$, $n = 4$ $\quad \sqrt[4]{6}\left(\cos\dfrac{3\pi}{16} + i\sin\dfrac{3\pi}{16}\right)$

$\sqrt[4]{6}\left(\cos\dfrac{11\pi}{16} + i\sin\dfrac{11\pi}{16}\right)$

$\sqrt[4]{6}\left(\cos\dfrac{19\pi}{16} + i\sin\dfrac{19\pi}{16}\right)$

$\sqrt[4]{6}\left(\cos\dfrac{27\pi}{16} + i\sin\dfrac{27\pi}{16}\right)$

Critical thinking questions:

23) Show that $-i\sqrt[23]{n}$ is a 23rd root of ni.

$\left(-i\sqrt[23]{n}\right)^{23} = (-i)^{23} \cdot \left(\sqrt[23]{n}\right)^{23} = in$

24) Solve for $x : (2 + i)x = 1 + 2i$ $\quad \dfrac{4}{5} + \dfrac{3}{5}i$

Hint: x is a complex number.