EQUATIONS INVOLVING RADICALS

1. Isolate the radicals
2. Get rid of the radical sign
3. If there is still a radical sign left, repeat steps 1 and 2.
4. Solve the remaining equation
5. Check for extraneous solutions

Note

1. If a value is an extraneous solution, it is not a solution to the original problem.
2. It is very important to check your results in the original equation. In many equations, one of the results may not satisfy the original equation. However, sometimes it is possible that all results that you have found will be acceptable.

EXAMPLE 42 Solve the radical equation $\sqrt{2x+5} = 7$.

EXAMPLE 43 Solve $\sqrt{2x-5} + x = 4$.

EXAMPLE 44 Solve $\sqrt{10x+56} - \sqrt{2x+8} = 4$.

EXAMPLE 45 Solve the equation $5 + \sqrt[3]{x+3} = 3$.

EXAMPLE 46 Solve the equation $\sqrt{4x+1}+\sqrt{x+2}=\sqrt{10x+5}$.

Check Yourself 11

Solve the equations.

1. $\sqrt{5x+3}=4$
2. $\sqrt{2x-4}=x-2$
3. $\sqrt{x+2}-\sqrt{4x+8}=-3$
4. $\sqrt{3x^2-2x+15}+\sqrt{3x^2-2x+8}=7$

Answers

1. $\frac{13}{5}$ 2. 2, 4 3. 7 4. $-\frac{1}{3}$, 1

INEQUALITIES

EXAMPLE 24 Solve the inequality $\sqrt{2x-1}<x+2$.

EXAMPLE 25 Solve the inequality $\sqrt{x^2+x-2}>x$.

3. $\sqrt{x^2-x}>1+x$

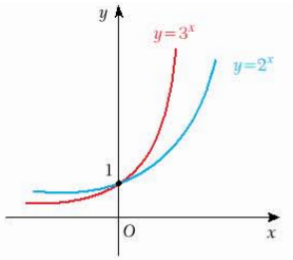
2 EXPONENTIAL FUNCTIONS

Definition basic exponential function

A function of the form $f(x) = a^x$ for a constant $a > 0$, $a \neq 1$ is called a basic exponential function with base a.

EXAMPLE 1 Determine which functions are exponential.

a. $p(x) = 2^{2x}$ b. $r(x) = 3^{-x}$ c. $s(x) = 5^{\frac{x}{4}}$ d. $t(x) = x^3$ e. $u(x) = (-3)^x$



A. EXPONENTIAL EQUATIONS

1. Equations of the Form $a^{f(x)} = a^{g(x)}$

EXAMPLE 1 Solve each equation for x.

a. $2^{3x} = 2^9$ b. $7^{x+9} = 7^{2x-15}$ c. $(\frac{3}{11})^{3x-10} = (\frac{11}{3})^{7x-10}$

2. Equations of the Form $a^{f(x)} = b$

EXAMPLE 2 Solve the equations.

a. $3^{x-1} = 4$ b. $2^{x^2} = 3$ c. $3^{2x-3} = -1$

3. Equations of the Form $a^{f(x)} = b^{g(x)}$

EXAMPLE 3 Solve the equations.

a. $2^{x+1} = 3^{x-2}$ b. $2^{x-1} \cdot 5^{x+3} = 3$

4. Working with Exponential Equations

EXAMPLE 4 Solve the equations.

a. $3^{2x} - 3^x = 72$ b. $4^x - 2^{x+4} + 48 = 0$

EXAMPLE 5 Solve $3^{3x} + 3^{1-3x} - 4 = 0$ for x.

EXAMPLE 7 Solve the equations.

a. $9^x + 4^x = \frac{5}{2} \cdot 6^x$ b. $(10 \cdot 25^{\frac{1}{x}}) + (10 \cdot 4^{\frac{1}{x}}) - (29 \cdot 10^{\frac{1}{x}}) = 0$

EXPONENTIAL INEQUALITIES

EXAMPLE 13 Solve the inequalities.

a. $2^{4x} < 16$ b. $3^x \cdot (\frac{1}{3})^{x-3} \leq (\frac{1}{27})^x$

EXAMPLE 14 Solve the inequalities.

a. $4^{\frac{5x-1}{2x-1}} \geq 64$ b. $9^x - 5 \cdot 3^x + 6 \leq 0$