
π futuRe maTh π
π maTh E1 E2 E3 π
x3
π maTh Level 1 & 2 π
3D Volume

Course Content: Choose your topic …

MATHS L1 to L2

Whole Number and Functions

place value

negative numbers

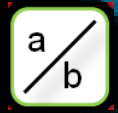

add and subtract


multiply divide

round numbers

ratio scale

Parts of a whole



fraction

decimal numbers


percent

percent decimal fraction

Measure and Shape

metric measure

imperial measure

perimeter

area

volume

formulae bodmas

Handling Data


charts data

averages


prob-ability

Topic Introduction : Volume

VOLUME

The volume of an object tells you how big the object is, how long, wide and high. All objects you can pick up, touch and move around have a volume no matter how thin they might be in any particular direction.

Volume is a measurement of the lengths along the sides of an object and multiplying these numbers tells you how many 'cubes' of 3D space fill it. Therefore the index number this time is 3. This method finds simple box shaped volumes but more complicated shapes such as cylinders and sphere require formulas to find their volumes.

Choose an icon to select where to start

Warm up Exercise 1

VOLUME

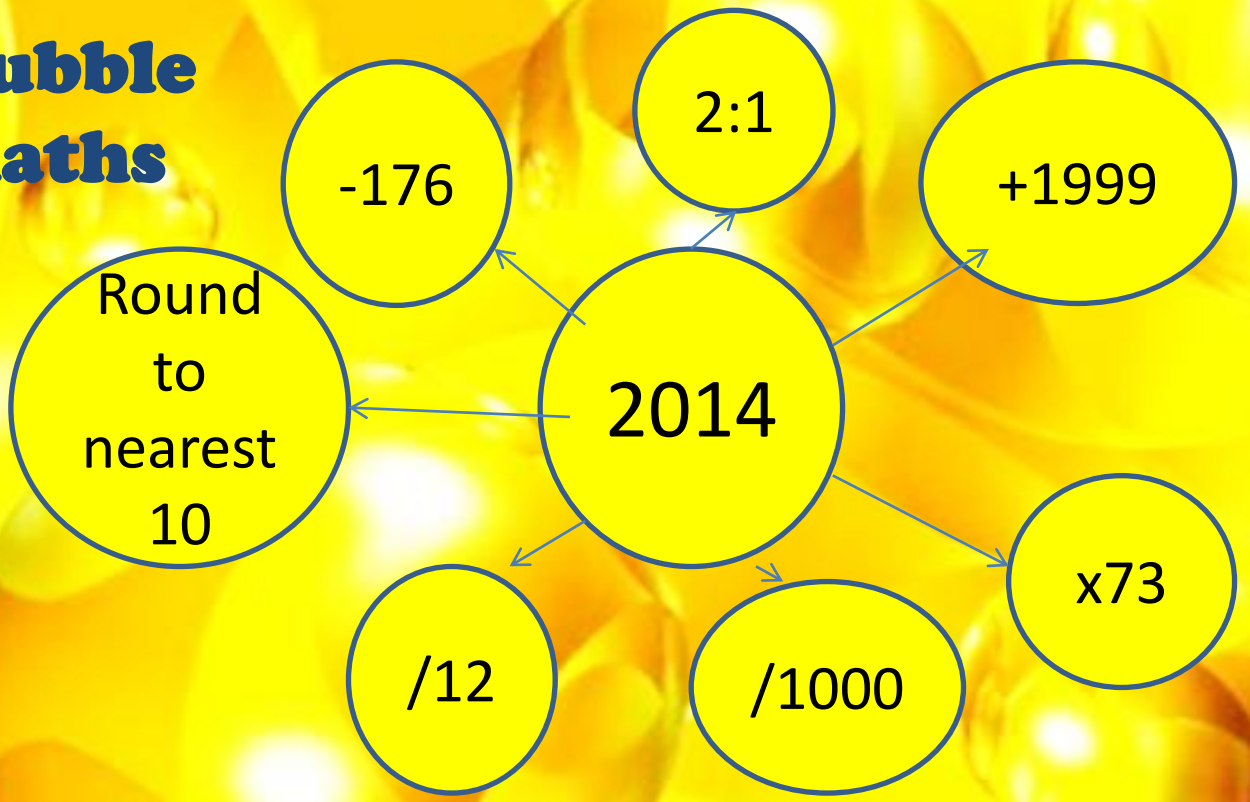
1 x 4 =	
2 x 4 =	
3 x 4 =	
4 x 4 =	
5 x 4 =	
6 x 4 =	
7 x 4 =	
8 x 4 =	
9 x 4 =	
10 x 4 =	

1	7	3	10	6	4	8	5	9	2
8									
7									
3									
10									
4									
2									
5									
6									
9									

Lets start today by revising ! Complete the above sums and multiplication grid

Warm up Exercise 2

Bubble maths



Calculate the instruction on the central number

Warm up Exercise 3

0.9mm=	.....cm
1.25m	.....cm
0.06m	.....mm
2.75km	.....m
71m	.....km
5cm	.....m
35mm	.....m
1345mm	.....cm
85ml	cl
1.08l	cl
0.004l	.....ml

0.004l	.....ml
40cl	l
900ml	l
101ml	cl
0.76g	.....mg
0.025kg	.....g
7200g	..... kg
56mg	.....g

Progress Checker 1

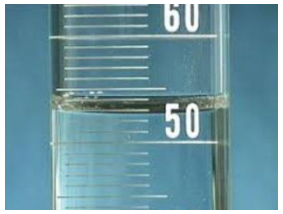
What do you already know about Volume ?

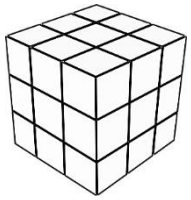
How would you rate your skills in finding Volumes of 3D objects ?

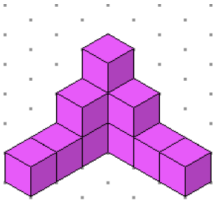
1) Excellent ability
2) Good ability, but working to improve
3) Ok, making a start but I know I have lots to still learn

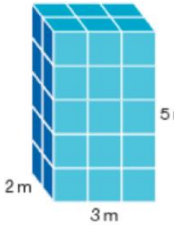
My aims for today Date: **are...**

A Find the volume of simple cube and cuboid shaped objects
B Find the volume of pyramids, cones, cylinders and spheres
C Use volume formulae and know the units for volume and solve practical problems









$V = l \times w \times h$
$V = 3\text{ m} \times 2\text{ m} \times 5\text{ m}$
$V = 30$ cubic meters

Introductory Video and Discussion

What does 'Volume' mean and what is it measuring ?

What are the units for the volume of a shape ?

How do you measure the volume of a liquid (not its capacity !) ?

What are 'Dimensions' and why are they important in the topic of volume ?

What is a prism and what is it about prisms that make it easier to find their volume ?

How can we find the volume of a sphere such as a planet, star ..?

Watch the introductory video and then discuss the above

Your thoughts..

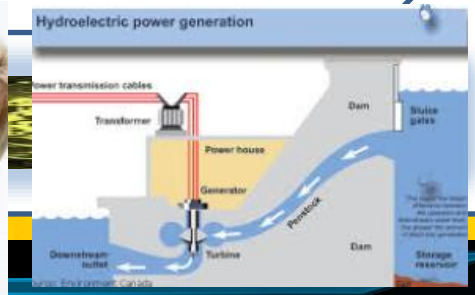
Vocabulary and Jobs

Volume
Space
3 Dimensions (3D)
Depth
Cubed
Cubic
Height
Solid
Net
Interior
Capacity
Formula
CC ml cm^3

These are the words you will be using in this topic

Builder
Packaging
Room/building Designer
Scuba Diver
Fireman
Nurse
Car Mechanic
Manufacturer
Water treatment
Powerplant
…. Can you think of more?
……
……



volume

- amount of space occupied by a 3D object, measured in cubic units.

EXAMPLE:  What do you think the volume of the yellow container would be in cm^3 ?

27 cm^3

CHECK

Well done !!

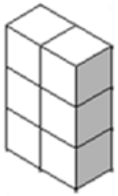
NOTE:
Different to capacity - the amount a container can hold

A block has volume - it takes up space.

A bottle has volume - it takes up space, and it has capacity - the amount it can hold.

Volume: how to find volumes

Just count the number of 'CUBES' you can see.. this is called the object's 'Volume'

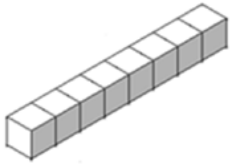


Here there are 6 cubes !

Here its 6 again !

Now there are 8 cubes !

How many are here???



Or here .... ??

Or here...??

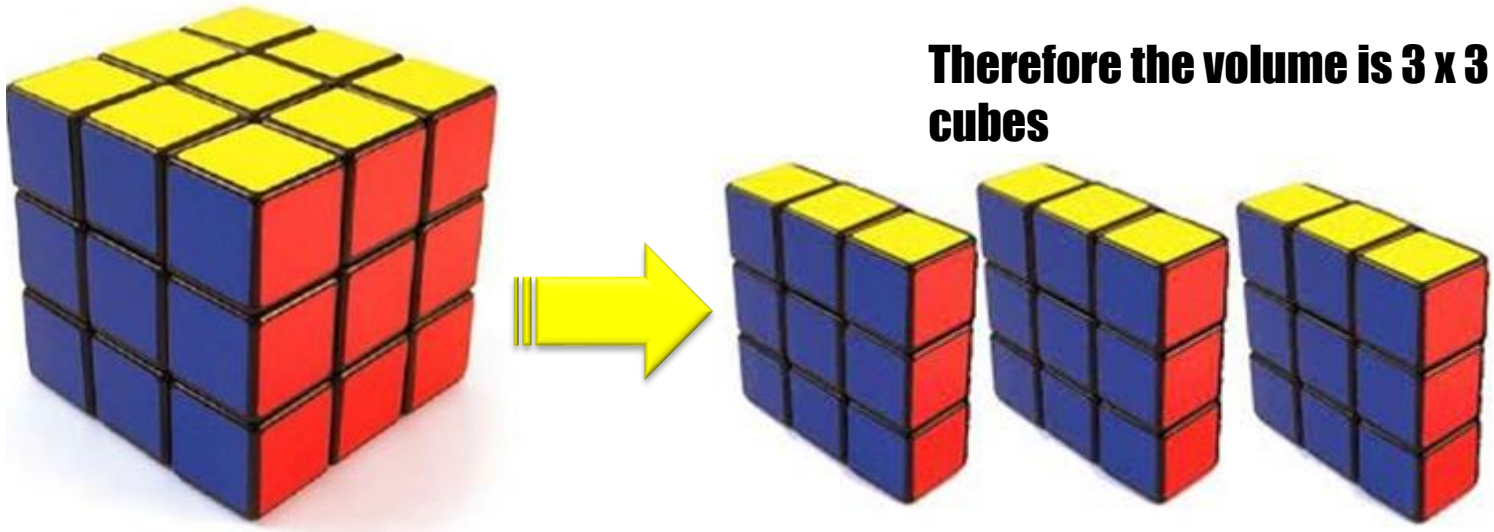
Volume: how to find volumes

If a 3D object is a **cube** or **cuboid** shape it can be easier to find the volume (number of cubes) not by counting them but by multiplying the **rows**, **columns** and **layers** of cubes.

Below is a cube but this is really made up of lots of layers of cubes on top of each other.

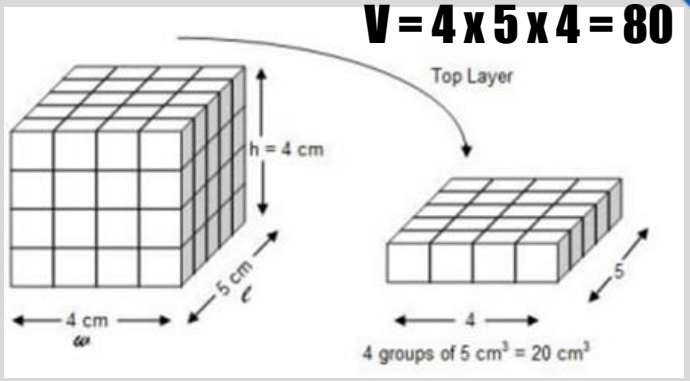
There are 3 cubes in a row... there are 3 rows.... There are 3 layers

Therefore the volume is 3 x 3 x 3 = 27 cubes

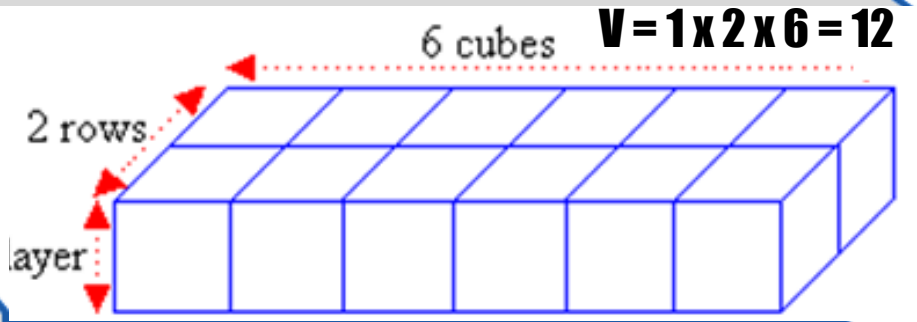


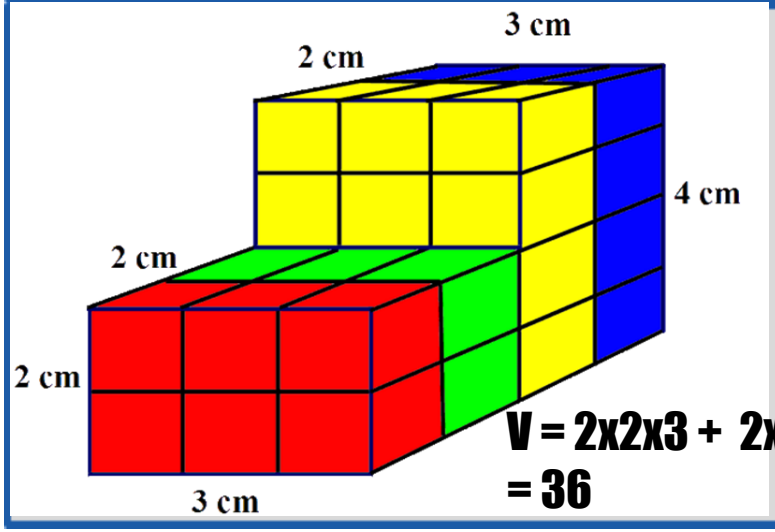
Lesson: Main Teach 3

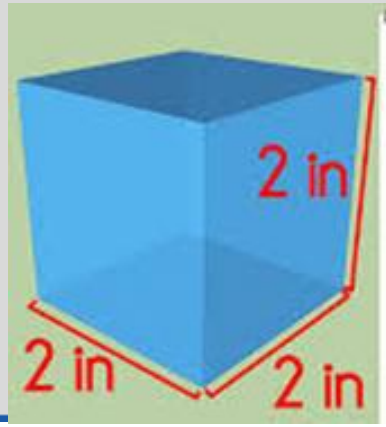
Volume: cuboid shaped examples V = Volume



Multiply the Rows x Columns x Layers



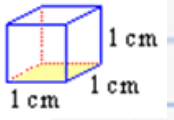




2 × 2 × 2
= 8

Lesson: Main Teach 4

Size of cubes…

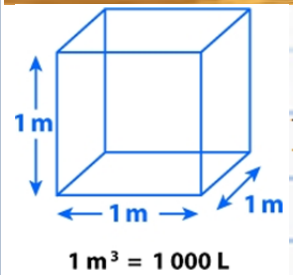


1 cubic centimetre =
1 cc = $1cm^3$ = 1 ml

Used for small volumes such as gas canisters, food containers, motorbike engines.

cm^3

=1ml



1 cubic metre =
1,000,000 cc
1,000 litres (1KL)

Used for large volumes such as Storage space, buildings, chemicals

m^3

=1000L or 1 KL

VOLUME

Lesson: Main Teach 5

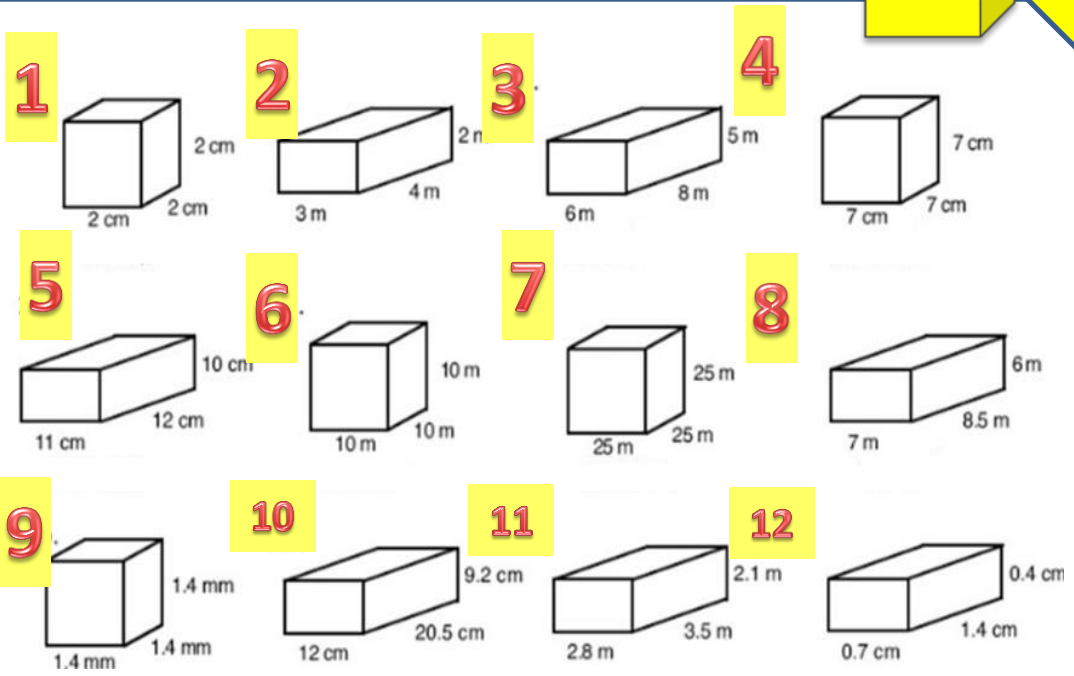
Cuboid volume examples....

Cube, side length 6cm
V=? V= 6x6x6=216cm^3

Cuboid W=10 L=6 H=4
V=? V= 10x6x4 = 240

Cube, side length 8m
V=? V= 8x8x8 = 512m^3

Cuboid W=2 L=? H=3
V=18 L=18/2/3 = 3



Can you find the volume of the above cuboid shapes by multiplying their lengths x widths x heights ?

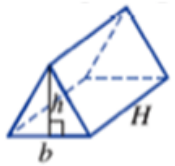
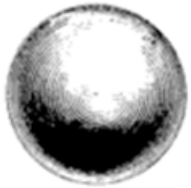
Don't forget at the end to put the unit (the size of the cube)...either cm^3 or m3... or even mm^3 !

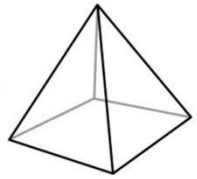
Lesson: Main Teach 6

Other shapes have a formula to be able to find their volume as the area changes through their height and so we cannot just multiply side lengths

To work out these volumes...

1) Write down their formula
2) Replace the letters in the formula with the numbers you know
3) Solve the sum to find V
4) Add the unit (eg cm^3)

	Rectangular Prism	Triangle prism	Sphere
Shape			
Volume:	$V = w \times l \times h$	$V=1/2\ bhH$	$V = \frac{4}{3}\pi r^3$

	Cylinder	Cone	Pyramid
Shape			
Volume:	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$	$V = 1/3$ LWH

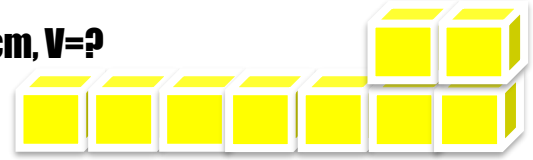
Lesson: Try out

Block 1 : Watch tutor led demo (in class or on video)

V = Volume (number of cubes inside a 3D object)

Try these, 1) A cuboid with length 10cm, width 3cm and height 5cm, V=?

2) 14 x 20 x 3 =

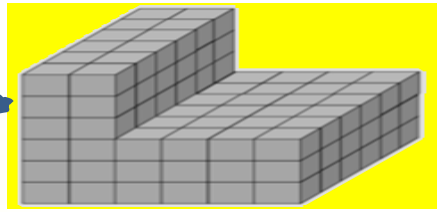
3)  How many cc?

Block 2 : Watch tutor led demo (in class or on video)

Try these, 4) Which is the largest cube? cm^3 M^3 mm^3 Km^3

5) Find the volume L = 2.3 W = 5.7 H = 9.1

6) V = ?



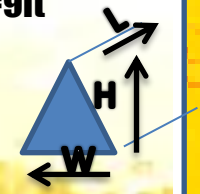
Block 3 : Watch tutor led demo (in class or on video)

Try these, 7) Cylinder volume with r=3cm L=8cm ?

8) Pyramid volume with L=4.5ft W=2 ft H=9ft

9) Cone volume is 30cm^3, r=5cm , length = ?

10) Triangle prism L=18, H=20 W=? V= 40



11) Sphere with radius 2,300m, V = ?

Lesson: Websites and links

Volume, shoot the cubes game	http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/VolumeShapesShoot.htm
Volumes of cuboids, adjust the length width and height and see cuboids	http://www.mathsisfun.com/flash.php?path=/geometry/images/cuboid-volume.swf&w=750&h=750&col=%23FFFFFF&title=Volume+of+a+Cuboid
Variety of volume videos on web page with formula and examples	http://www.onlinemathlearning.com/volume-formula.html
3D objects and their volumes web page	http://www.mathwarehouse.com/solid-geometry/
Create and animate cuboids of various sizes	http://illuminations.nctm.org/ActivityDetail.aspx?ID=6
Video showing how the formula is derived for a sphere (hard!)	http://www.youtube.com/watch?v=xuPl_8o_j7k
Great website with volume of solids lessons	http://www.mathexpression.com/volume-of-solids.html

Lesson: Practice – just the numbers

Find the volume of the Cuboids

$V = LWH$

1) L = 8cm, W = 2cm, H = 10cm V = ?
2) L = 5m, W = 3m, H = 9m V = ?
3) L = 6km, W = 5km, H = 2km V = ?
4) L = 11mm, W = 5mm, H = 2mm V = ?
5) L = 4.5ft, W = 5ft, H = 2ft V = ?
6) L = 0.1in, W = 5in, H = 2in V = ?
7) L = 50miles, W = 5miles, H = 2miles V = ?
8) L = 1/2cm, W = 5cm, H = 2cm V = ?
9) L = 30m, W = 5m, H = 250cm V = ?
10) L = 78cm, W = 5cm, H = 2m V = ?

Find the volume of the Cubes

$V = L^3$

11) L = 4, W = 4, H = 4 V = ?
12) L = 9, W = 9, H = 9 V = ?
13) L = 3in, W = 3in, H = 3in V = ?
14) L = 1.5m, W = 1.5m, H = 1.5m V = ?
15) L = 0.2cm, W = 0.2cm, H = 0.2cm V = ?
16) L = 10km, W = 10km, H = 10km V = ?

Find the volume of the Pyramids

$V = 1/3\ LWH$

1) L = 3, W = 2, H = 4 V = ?
2) L = 7, W = 1, H = 1 V = ?
3) L = 9, W = 5, H = 8 V = ?
4) L = 0.5, W = 3, H = 2 V = ?
5) L = 6, W = 3, H = 2.4 V = ?
6) L = 800cm, W = 200cm, H = 3m V = ?
7) L = 1mm, W = 1mm, H = 3mm V = ?
8) L = 2.3km, W = 1.9km, H = 1km V = ?
9) L = 6in, W = 2in, H = 1 ft V = ?

Find the volume of the Cones

$V = 1/3\ \pi\ r^2 L$

10) r=9 L= 3 V = ?
11) r=0.5 L= 5 V = ?
12) r=100cm L= 2m V = ?
13) d=7 L=16 V = ?
14) d=3.4 in L=8 in V = ?
15) d=26 L=30 V = ?

Find the volume of the Spheres

$V = 4/3\ \pi\ R^3$

1) r=45cm V = ?
2) r=1 mile V = ?
3) r=17in V = ?
4) r=80ft V = ?
5) r=20cm V = ?
6) r=90m V = ?
7) r=0.5cm V = ?
8) r=6mm V = ?
9) r=9.2 ft V = ?

Find the volume of the Cylinders

$V = \pi\ r^2\ L$

10) L = 2cm, r = 5cm V = ?
11) L = 7m, r = 18m V = ?
12) L = 5 ft, r = 5 ft V = ?
13) L = 90, r = 20 V = ?
14) L = 1 km, r = 4m V = ?
15) L = 2miles, r = 30ft V = ?

Lesson: Practice – word problems

1) A tunnel through a mountain is half of a cylinder shape with the tunnel height of seven metres and the tunnel length of half a kilometre long. What is the volume of rock that was removed from the mountain to create the tunnel (in m^3) ?

2) A football is inflated with 6000cm^3 of air. What radius does the ball inflate to?

3) An office block advertises its 'To-let' space as having 40,000 ft^3. With a building length of 20m and width of 20m, how tall is the building if it is a cuboid shape?

4) A container crate measures 6m x 3m x 3m. What volume of products can fill a container?

5) A funnel in a factory is filled with hot, melted chocolate during the production of sweets. If the funnel has a diameter of 2m and contains 2m^3 of chocolate, how tall is the funnel?

6) The pyramids of Egypt are 800m long and wide, having a square base and has a height of 139m. Work out the total volume of the blocks that make up the pyramid.

Lesson: Practice – Making it Functional 1

- A warehouse is organising delivery of boxes containing a variety of food items. They are distributing them all over the country to supermarkets.
- The distribution lorry is 2.25m wide, 2.2 m high and 6m long.

- The boxes containing the food stuffs are 45cms wide, 30cms high and 50cms long.

Lesson: Practice – Making it Functional 2

Use the information on the previous page

- 1.a) What do you need to do to find how many boxes will fit on a lorry?
- b) Calculate how many boxes can be loaded onto a lorry. Show your working.
- c) The warehouse has to deliver 2000 boxes of food this week. How many deliveries will they have to make?

- The boxes contain different foodstuffs. 7% of the boxes being delivered contain baked beans. The tins themselves have an 8cm diameter and are 11cm high.
- **2 a) Calculate the maximum amount of tins of beans that can be packed in a box. Show your calculations.**
- **b) How many boxes will you fill?**
- **c) You have a weight limit for each box. The tins of beans weigh 20 grams per 40cm cubed. How much will each tin weigh to 2 d.p.s? (pi = 3.14)**
- **d) The weight limit for each box is 25kg. Will you have to reduce the number of tins packed in the box?**
- **e) Why do you think there might be a weight limit for the boxes?**
- **(Calculators may be used for these questions)**

TOPIC ANSWERS 1

VOLUME

Block 1 answers

1) 150 cm3
2) 840
3) 9 cc

Block 2 answers

4) km3
5) 119.301
6) 144 cubes

Block 3 answers

7) 226.08 cm3
8) 27 ft3
9) 1.15cm
10) 0.22
11) 50.9 B approx

Find the volume of the Cuboids answers

1) 160cm3
2) 135 m3
3) 70 km3
4) 110 mm3
5) 45 ft3
6) 1 in3
7) 500 cm3
8) 5 cm3
9) 37.5 m3
10) 780 cm3

Find the volume of the Cuboids answers

11) 64
12) 729
13) 27 in3
14) 3.375 m3
15) 0.008 cm3
16) 1000 km3

Find the volume of the Pyramids answers

1) 8
2) 2.33
3) 120
4) 1
5) 14.4
6) 16 m3
7) 1 mm3
8) 1.46 km3
9) 48 in3

Find the volume of the Cones answers

10) 254.34
11) 1.31
12) 2.09 m3
13) 205.25
14) 24.2 in3
15) 5309.29

Find the volume of the Spheres

1) 381,703.5 cm3 (38.2 m3)
2) 4.2 miles3
3) 20579.5 in3
4) 2,144,660 ft3
5) 33,510 cm3
6) 3,053,628 m3
7) 0.5 cm 3
8) 904.8 mm3
9) 3,261.8 ft3

Find the volume of the Cylinders

10) 62.8 cm3
11) 7125 m3
12) 392.7 ft3
13) 113,097
14) 50,265.5 m3
15) 9,952,565.5 ft3

TOPIC ANSWERS 2

Word Problem answers

1) $0.5 \times 3.14 \times r^2 \times L$
$= 0.5 \times 3.14 \times 7^2 \times$
$500m = 38{,}484.5m^3$

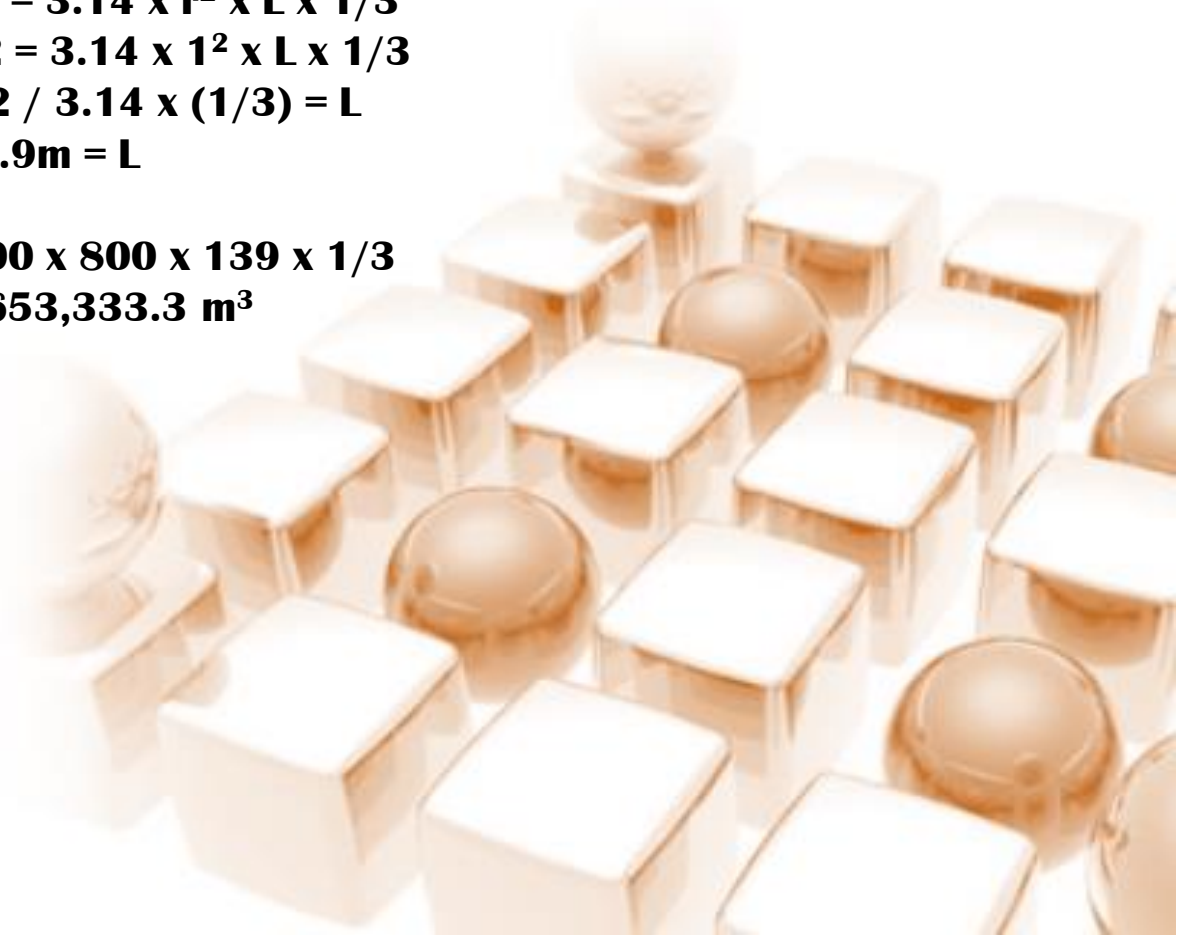
2) $V=3.14 \times r3$
so.. $6000= 3.14 \times r3$
$6000/3.14 = r^3$
$\sqrt[3]{1910.8} = r$
$12.4cm = r$

3) $40000 = l\ w\ h$
so.. $40000/20x20 = h$
$100m = h$

4) $6 \times 3 \times 3 = 54m^3$

5) $V = 3.14 \times r^2 \times L \times 1/3$
so.. $2 = 3.14 \times 1^2 \times L \times 1/3$
$2 / 3.14 \times (1/3) = L$
$1.9m = L$

6) $800 \times 800 \times 139 \times 1/3$
$= 29{,}653{,}333.3\ m^3$

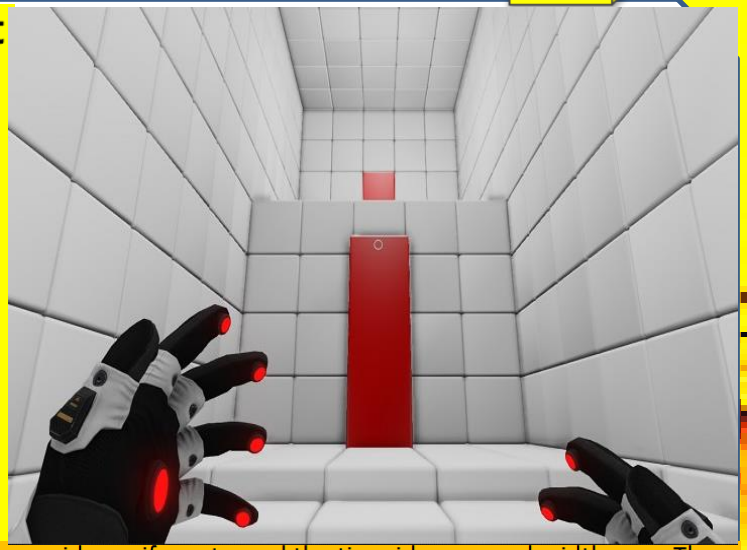


TOPIC ANSWERS 3

- 1.a) Find how many boxes will fit into the **width, height and length** of the lorry and **multiply the 3 totals**.

- b) Width 2.25m (225cm) / 45cm = 5.
- Height 2.2m (220cm) / 30cm = 7 (10cm left over)
- Length 6m (600cm) / 50cm = 12.
- 5 x 7 x 12 boxes = **420 boxes on a lorry**.

- c) **Total of 5 deliveries**. 4 deliveries can distribute 420 boxes each, and the 5th delivery can put on the remaining 320 boxes.



- 2a) Boxes are 45cm wide, so if you turned the tins sideways and widthways. The tins would be 30/8 = 3, 45/11 = 4 and 50/8 =6.
- **3 x 4 x 6 = 72 tins maximum.**

- If you packed them the other possible ways you would only fit in 60 tins.
- b) 7% of 2000 boxes = 140. You could fit them into **2 boxes.**

- c) The volume for each tin is Pi r squared x height. 4 x 4 x 3.14=50.24cm squared x 11cm = 552.64cm cubed. 552.64 x 20g/40cm cubed = **Each tin weighs 276.32 grams**.

- d) Tin =276.32g x 72 tins maximum = 19895g. 19895/1000 = 19.895kg. **No the number of tins does not need reducing.**

- e) Any sensible answers. Maybe the boxes would be **too heavy to be lifted safely**, or the boxes will be more prone to **splitting with too much weight.**

Progress Checker 2

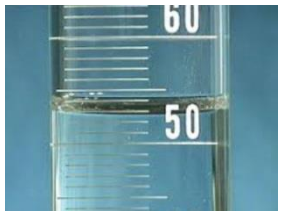
What do you now know about Volume ? WHAT DID YOU LEARN? Write some examples…

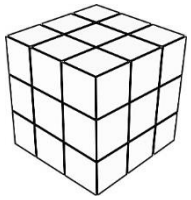
How would you now rate your skills in finding Volumes of 3D objects ?

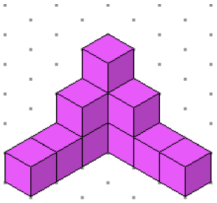
1) Excellent ability
2) Good ability, but working to improve
3) Ok, making a start but I know I have lots to still learn

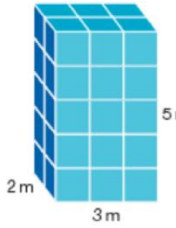
My aims for today Date: **were…**

A Find the volume of simple cube and cuboid shaped objects
B Find the volume of pyramids, cones, cylinders and spheres
C Use volume formulae and know the units for volume and solve practical problems









$V = l \times w \times h$
$V = 3\text{ m} \times 2\text{ m} \times 5\text{ m}$
$V = 30$ cubic meters

Continuing to Study and Learn

What else can you do to help yourself to learn and practice? Here are ten suggestions, record which you do each week and also record your progress.

Internet websites
Repeat the lesson, make notes, organise a folder, revise

Own maths workbook
Study together with a friend or family member

Finish activities in this book
Complete class handouts or tasks

Practice exams / past papers
Use maths skills learnt at home or at work in real situations

Play games
Experiment yourself, try new things ask yourself questions

Try making a graph of number of practice methods you use against your progress score in each topic. Are you showing more practice gives better results?