Questions

Q1.

The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

(4)

(Total for question = 4 marks)

Q2.

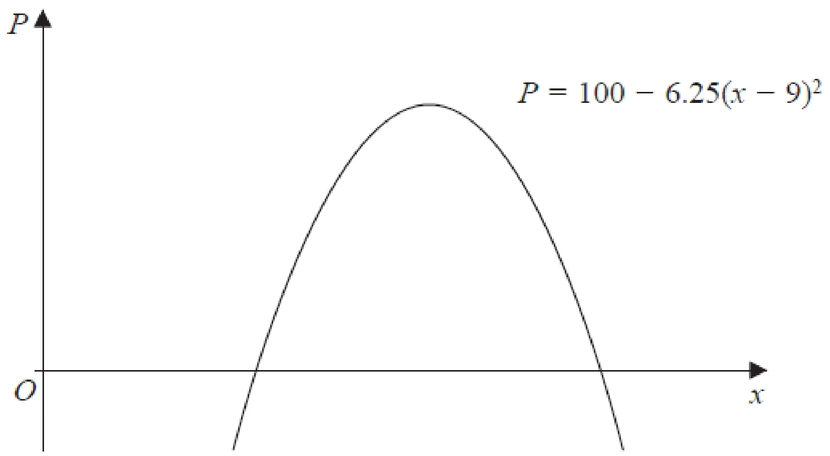


Figure 1

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where P is the profit measured in thousands of pounds and x is the selling price of the toy in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

(a) explain why £15 is not a sensible selling price for the toy.

(2)

Give that the company made an annual profit of more than £80 000

(b) find, according to the model, the least possible selling price for the toy.

(3)

The company wishes to maximise its annual profit.

State, according to the model,

(c) (i) the maximum possible annual profit,

(ii) the selling price of the toy that maximises the annual profit.

(2)

(Total for question = 7 marks)

Q3.

A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where T tonnes is the total mass of tin mined in the n years after the start of mining.

Using this model,

(a) calculate the mass of tin that will be mined up to 1st January 2020,

(1)

(b) deduce the maximum total mass of tin that could be mined,

(1)

(c) calculate the mass of tin that will be mined in 2023.

(2)

(d) State, giving reasons, the limitation on the values of n.

(2)

(Total for question = 6 marks)

Q4.

In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0$$

(3)

(b) Hence find all real solutions of

$$3(y - 2)^6 - 17(y - 2)^4 - 6(y - 2)^2 = 0$$

(3)

(Total for question = 6 marks)

Q5.

A curve C has equation $y = f(x)$ where

$$f(x) = -3x^2 + 12x + 8$$

(a) Write f(x) in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The curve C has a maximum turning point at M.

(b) Find the coordinates of M.

(2)

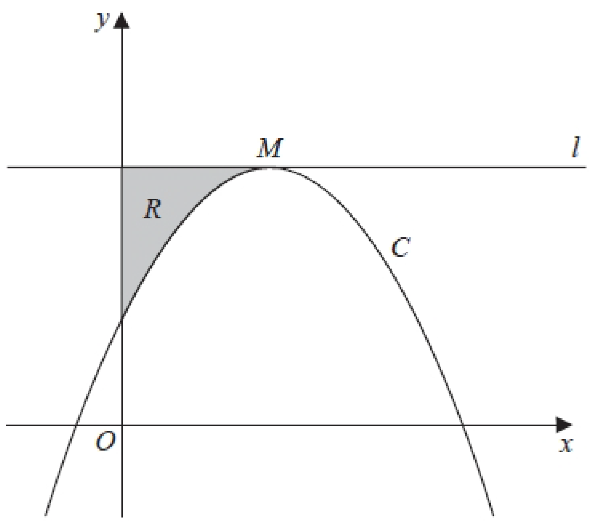


Figure 3

Figure 3 shows a sketch of the curve C.

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Using algebraic integration, find the area of R.

(5)

(Total for question = 10 marks)

Q6.

An archer shoots an arrow.

The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \quad d \geq 0$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.

(3)

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.

(1)

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form

$$A - B(d - C)^2$$

where A, B and C are constants to be found.

(3)

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \quad d \geq 0$$

Hence or otherwise, find, for the adapted model

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.

(2)

(Total for question = 9 marks)

Q7.

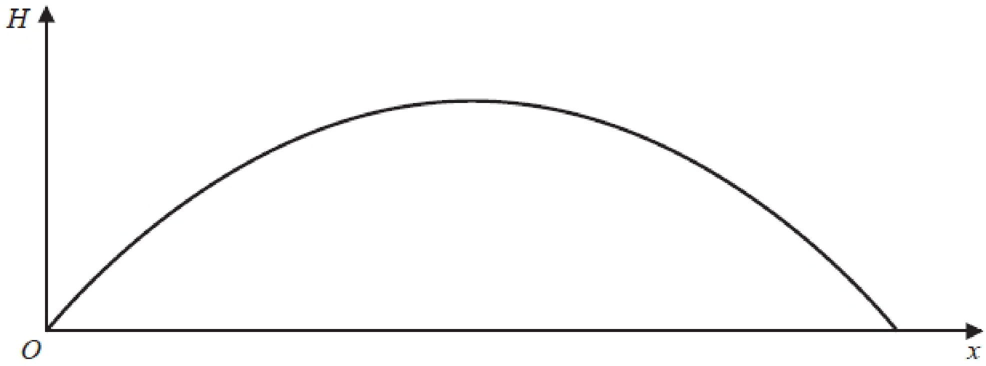


Figure 1

Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground, H metres, has been plotted against the horizontal distance, x metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking H with x that models this situation.

(3)

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from O.

(3)

(c) Give one limitation of the model.

(1)

(Total for question = 7 marks)

Q8.

$$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write f(x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation y = f(x) showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation y = f(x) onto the curve with equation y = g(x) where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

(Total for question = 10 marks)

Q9.

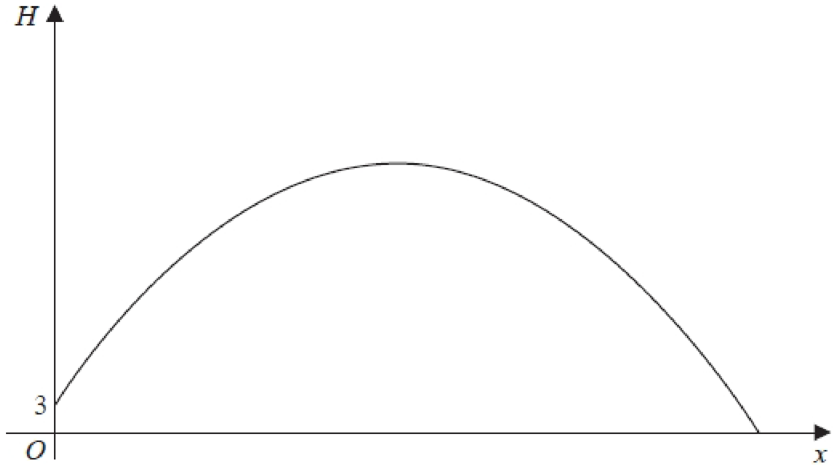


Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that H is modelled as a **quadratic** function in x

(a) find H in terms of x

(5)

(b) Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,
(ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model.
Give one other limitation of this model.

(1)

(Total for question = 9 marks)

Q10.

Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

(Total for question = 8 marks)

Q11.

The curve C has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

and a and b are constants.

Given

- the point (2, 10) lies on C
- the gradient of the curve at (2, 10) is -3

(a) (i) show that the value of a is -2

(ii) find the value of b.

(4)

(b) Hence show that C has no stationary points.

(3)

(c) Write $f(x)$ in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic expression to be found.

(2)

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = f(0.2x)$$

and the coordinate axes.

(2)

(Total for question = 11 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	Realises that $k = 0$ will give no real roots as equation becomes 3 = 0 (proof by contradiction)	B1	3.1a
	(For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1	2.4
	$4k(4k-3) < 0$ with attempt at solution	M1	1.1b
	So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leqslant k < \frac{3}{4}$ *	A1*	2.1
			(4 marks)

Notes

B1 : Explains why $k = 0$ gives no real roots
M1 : Considers discriminant to give quadratic inequality – does not need the $k \neq 0$ for this mark
M1 : Attempts solution of quadratic inequality
A1* : Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

Q2.

Question	Scheme	Marks	AOs
(a)	Attempts $P = 100 - 6.25(15-9)^2$	M1	3.4
	$= -125$ $\therefore$ not sensible as the company would make a loss	A1	2.4
		(2)	
(b)	Uses $P > 80 \Rightarrow (x-9)^2 < 3.2$ or $P = 80 \Rightarrow (x-9)^2 = 3.2$	M1	3.1b
	$\Rightarrow 9 - \sqrt{3.2} < x < 9 + \sqrt{3.2}$	dM1	1.1b
	Minimum Price = £7.22	A1	3.2a
		(3)	
(c)	States (i) maximum profit =£ 100 000 and (ii) selling price £9	B1 B1	3.2a 2.2a
		(2)	
			(7 marks)

(a)

M1: Substitutes $x = 15$ into $P = 100 - 6.25(x-9)^2$ and attempts to calculate. This is implied by an answer of -125. Some candidates may have attempted to multiply out the brackets before they substitute in the $x = 15$. This is acceptable as long as the function obtained is quadratic. There must be a calculation seen or implied by the value of -125.

A1: Finds $P = -125$ or states that $P < 0$ **and** explains that (this is not sensible as) the company would make a loss.

Condone $P = -125$ followed by an explanation that it is not sensible as the company would make a loss of £125 rather than £125 000. An explanation that it is not sensible as "the profit cannot be negative", "the profit is negative" or "the company will not make any money", "they might make a loss" is incomplete/incorrect. You may ignore any misconceptions or reference to the price of the toy being too cheap for this mark.

Alt: **M1:** Sets $P = 0$ and finds $x = 5, 13$ A1: States $15 > 13$ and states makes a loss

(b)

M1: Uses $P ... 80$ where ... is any inequality or "=" in $P = 100 - 6.25(x-9)^2$ and proceeds to $(x-9)^2 ... k$ where $k > 0$ and ... is any inequality or "="

Eg. Condone $P < 80$ in $P = 100 - 6.25(x-9)^2 \Rightarrow (x-9)^2 < k$ where $k > 0$ If the candidate attempts to multiply out then allow when they achieve a form $ax^2 + bx + c = 0$

dM1: Award for solving to find the two positive values for x. Allow decimal answers

FYI correct answers are $\Rightarrow 9 - \sqrt{3.2} < x < 9 + \sqrt{3.2}$ Accept $\Rightarrow x = 9 \pm \sqrt{3.2}$

Condone incorrect inequality work $100 - 6.25(x-9)^2 > 80 \Rightarrow (x-9)^2 > 3.2 \Rightarrow x > 9 \pm \sqrt{3.2}$

Alternatively award if the candidate selects the lower of their two positive values $9 - \sqrt{3.2}$

A1: Deduces that the minimum Price = £7.22 (£7.21 is not acceptable)

(c)

(i) B1: Maximum Profit = £ 100 000 with units. Accept 100 thousand pound(s).

(ii) B1: Selling price = £9 with units

SC 1: Missing units in (b) and (c) only penalise once in these parts, withhold the final mark.

SC 2: If the answers to (c) are both correct, but in the wrong order score SC B1 B0

If (i) and (ii) are not written out score in the order given.

Q3.

Question	Scheme	Marks	AOs
(a)	117 tonnes	B1	3.4
		(1)	
(b)	1200 tonnes	B1	2.2a
		(1)	
(c)	Attempts $\left\{1200-3\times(5-20)^2\right\}-\left\{1200-3\times(4-20)^2\right\}$	M1	3.1a
	93 tonnes	A1	1.1b
		(2)	
(d)	States the model is only valid for values of n such that $n \le 20$	B1	3.5b
	States that the total amount mined cannot decrease	B1	2.3
		(2)	
			(6 marks)

Notes

Note: Only withhold the mark for a lack of tonnes, once, the first time that it occurs.

(a)
B1: 117 tonnes or 117 t.
(b)
B1: 1200 tonnes or 1200 t.
(c)
M1: Attempts $T_5 - T_4 = \left\{1200-3\times(5-20)^2\right\}-\left\{1200-3\times(4-20)^2\right\}$ May be implied by $525-432$

Condone for this mark an attempt at $T_4 - T_3 = \left\{1200-3\times(4-20)^2\right\}-\left\{1200-3\times(3-20)^2\right\}$

A1: 93 tonnes or 93 t
(d)
For one mark

Shows an appreciation of the model

- States $n \le 20$ or $n < 20$
- Condone for one mark $n \le 40$ or $n < 40$ **with** "the mass of tin mined cannot be negative" oe
- Condone for one mark $n = 40$ **with** a statement that "the mass of tin mined becomes 0" oe
- after 20 years the (total) amount of tin mined starts to go down (n may not be mentioned and total may be missing)
- after 20 years the (total) mass reaches a maximum value. (Similar to above)
- States T_{max} is reached when $n = 20$

For two marks

States the limitation on n and explains fully. (Total mass, not mass must be used)

- States that $n \le 20$ and explains that the total mass of tin cannot decrease.
- Alternatively states that n cannot be more than 20 and the total mass of tin would be decreasing
- $0 < n \le 20$ as the maximum total amount of tin mined is reached at 20 years

Q4.

Question	Scheme	Marks	AOs
(a)	$3x^3-17x^2-6x=0\Rightarrow x(3x^2-17x-6)=0$	M1	1.1a
	$\Rightarrow x(3x+1)(x-6)=0$	dM1	1.1b
	$\Rightarrow x=0,-\frac{1}{3},6$	A1	1.1b
		(3)	
(b)	Attempts to solve $(y-2)^2=n$ where n is any solution $\geqslant 0$ to (a)	M1	2.2a
	Two of $2, 2\pm\sqrt{6}$	A1ft	1.1b
	All three of $2, 2\pm\sqrt{6}$	A1	2.1
		(3)	
		(6 marks)	

Notes

(a)

M1: Factorises out or cancels by x to form a quadratic equation.

dM1: Scored for an attempt to find x. May be awarded for factorisation of the quadratic or use of the quadratic formula.

A1: $x=0,-\frac{1}{3},6$ and no extras

(b)

M1: Attempts to solve $(y-2)^2=n$ where n is any solution $\geqslant 0$ to (a). At least one stage of working must be seen to award this mark. Eg $(y-2)^2=0\Rightarrow y=2$

A1ft: Two of $2, 2\pm\sqrt{6}$ but follow through on $(y-2)^2=n\Rightarrow y=2\pm\sqrt{n}$ where n is a positive solution to part (a). (Provided M1 has been scored)

A1: All three of $2, 2\pm\sqrt{6}$ and no extra solutions. (Provided M1A1 has been scored)

Q5.

Question	Scheme	Marks	AOs
(a)	$\mathrm{f}(x) = -3x^2 + 12x + 8 = -3(x \pm 2)^2 + \ldots$	M1	1.1b
	$= -3(x-2)^2 + \ldots$	A1	1.1b
	$= -3(x-2)^2 + 20$	A1	1.1b
		(3)	
(b)	Coordinates of $M = (2, 20)$	B1ft B1ft	1.1b 2.2a
		(2)	
(c)	$\int -3x^2 + 12x + 8 \, \mathrm{d}x = -x^3 + 6x^2 + 8x$	M1 A1	1.1b 1.1b
	Method to find $R =$ their $2 \times 20 - \int_0^2 \left(-3x^2 + 12x + 8\right) \mathrm{d}x$	M1	3.1a
	$R = 40 - \left[-2^3 + 24 + 16\right]$	dM1	1.1b
	$= 8$	A1	1.1b
		(5)	
	(10 marks)		
Alt(c)	$\int 3x^2 - 12x + 12 \, \mathrm{d}x = x^3 - 6x^2 + 12x$	M1 A1	1.1b 1.1b
	Method to find $R = \int_0^2 3x^2 - 12x + 12 \, \mathrm{d}x$	M1	3.1a
	$R = 2^3 - 6 \times 2^2 + 12 \times 2$	dM1	1.1b
	$= 8$	A1	1.1b

Notes:

(a)

M1: Attempts to take out a common factor and complete the square. Award for $-3(x\pm 2)^2+...$
Alternatively attempt to compare $-3x^2+12x+8$ to $ax^2+2abx+ab^2+c$ to find values of a and b

A1: Proceeds to a form $-3(x-2)^2+...$ or via comparison finds $a=-3, b=-2$

A1: $-3(x-2)^2+20$

(b)

B1ft: One correct coordinate

B1ft: Correct coordinates. Allow as $x=..., y=...$
Follow through on their $(-b, c)$

(c)

M1: Attempts to integrate. Award for any correct index

A1: $\int -3x^2+12x+8\,dx = -x^3+6x^2+8x\ (+c)$ (which may be unsimplified)

M1: Method to find area of R. Look for their $2\times"20"-\int_0^{"2"} f(x)\,dx$

dM1: Correct application of limits on their integrated function. Their 2 must be used

A1: Shows that area of $R = 8$

Q6.

Question	Scheme	Marks	AOs
(a)	Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$	M1	3.4
	Solves using an appropriate method, for example $d=\dfrac{-0.4\pm\sqrt{(0.4)^2-4(-0.002)(1.8)}}{2\times-0.002}$	dM1	1.1b
	Distance = awrt $204(\text{m})$ only	A1	2.2a
		(3)	
(b)	States the initial height of the arrow above the ground.	B1	3.4
		(1)	
(c)	$1.8+0.4d-0.002d^2=-0.002\left(d^2-200d\right)+1.8$	M1	1.1b
	$=-0.002\left((d-100)^2-10000\right)+1.8$	M1	1.1b
	$=21.8-0.002(d-100)^2$	A1	1.1b
		(3)	
(d)	(i) 22.1 metres	B1ft	3.4
	(ii) 100 metres	B1ft	3.4
		(2)	
		(9 marks)	

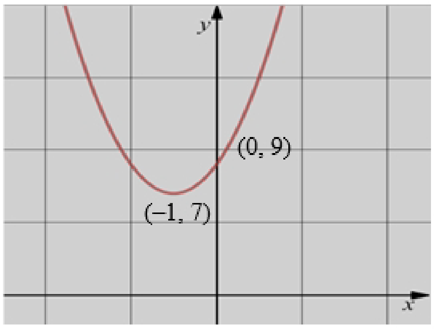
Notes:
(a) **M1:** Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$ **M1:** Solves using formula, which if stated must be correct, by completing square (look for $(d-100)^2=10900 \Rightarrow d=..$) or even allow answers coming from a graphical calculator **A1:** Awrt 204 m only
(b) **B1:** States it is the initial height of the arrow above the ground. Do not allow " it is the height of the archer"
(c) **M1:** Score for taking out a common factor of -0.002 from at least the d^2 and d terms **M1:** For completing the square for their $\left(d^2-200d\right)$ term **A1:** $=21.8-0.002(d-100)^2$ or exact equivalent
(d) **B1ft:** For their '21.8+0.3' =22.1m **B1ft:** For their 100m

Q7.

Question	Scheme	Marks	AOs
(a) Way 1	$H = Ax(40-x)$ {or $H = Ax(x-40)$ }	M1	3.3
	$x = 20,\ H = 12 \Rightarrow 12 = A(20)(40-20) \Rightarrow A = \frac{3}{100}$	dM1	3.1b
	$H = \frac{3}{100}x(40-x)$ or $H = -\frac{3}{100}x(x-40)$	A1	1.1b
		(3)	
(a) Way 2	$H = 12-\lambda(x-20)^2$ {or $H = 12+\lambda(x-20)^2$ }	M1	3.3
	$x = 40,\ H = 0 \Rightarrow 0 = 12-\lambda(40-20)^2 \Rightarrow \lambda = \frac{3}{100}$	dM1	3.1b
	$H = 12-\frac{3}{100}(x-20)^2$	A1	1.1b
		(3)	
(a) Way 3	$H = ax^2+bx+c$ (or deduces $H = ax^2+bx$) **Both** $x = 0,\ H = 0 \Rightarrow 0 = 0+0+c \Rightarrow c = 0$ **and either** $x = 40,\ H = 0 \Rightarrow 0 = 1600a+40b$ **or** $x = 20,\ H = 12 \Rightarrow 12 = 400a+20b$ **or** $\frac{-b}{2a} = 20$ $\{\Rightarrow b = -40a\}$	M1	3.3
	$b = -40a \Rightarrow 12 = 400a+20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2+1.2x$	A1	1.1b
		(3)	
(b)	$\{H = 3 \Rightarrow\}\ 3 = \frac{3}{100}x(40-x) \Rightarrow x^2-40x+100 = 0$ or $\{H = 3 \Rightarrow\}\ 3 = 12-\frac{3}{100}(x-20)^2 \Rightarrow (x-20)^2 = 300$	M1	3.4
	e.g. $x = \frac{40 \pm \sqrt{1600-4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	$\{$chooses $20+\sqrt{300} \Rightarrow\}$ greatest distance = awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
		(7 marks)	

Notes for Question	
(a)	
M1:	Translates the situation given into a suitable equation for the model. E.g. **Way 1:** {Uses $(0, 0)$ and $(40, 0)$ to write} $H = Ax(40-x)$ o.e. {or $H = Ax(x-40)$ }
	Way 2: {Uses $(20, 12)$ to write} $H = 12-\lambda(x-20)^2$ or $H = 12+\lambda(x-20)^2$
	Way 3: Writes $H = ax^2+bx+c$, and uses $(0, 0)$ to deduce $c=0$ and an attempt at using either $(40, 0)$ or $(20, 12)$ **Special Case:** Allow SC M1dM0A0 for not deducing $c=0$ but attempting to apply both $(40, 0)$ and $(20, 12)$
dM1:	Applies a complete strategy with appropriate constraints to find all constants in their model. **Way 1:** Uses $(20, 12)$ on their model and finds $A = ...$ **Way 2:** Uses either $(40, 0)$ or $(0, 0)$ on their model to find $\lambda = ...$ **Way 3:** Uses $(40, 0)$ and $(20, 12)$ on their model to find $a = ...$ and $b = ...$
A1:	Finds a correct equation linking H to x E.g. $H = \frac{3}{100}x(40-x)$, $H = 12-\frac{3}{100}(x-20)^2$ or $H = -0.03x^2+1.2x$
Note:	Condone writing y in place of H for the M1 and dM1 marks.
Note:	Give final A0 for $y = -0.03x^2+1.2x$
Note:	Give special case M1dM0A0 for writing down any of $H = 12-(x-20)^2$ or $H = x(40-x)$ or $H = x(x-40)$
Note:	Give M1 dM1 for finding $-0.03x^2+1.2x$ or $a=-0.03, b=1.2, c=0$ in an implied ax^2+bx or ax^2+bx+c (with no indication of $H = ...$)
(b)	
M1:	Substitutes $H=3$ into their quadratic equation and proceeds to obtain a 3TQ or a quadratic in the form $(x\pm\alpha)^2 = \beta$; $\alpha, \beta \neq 0$
Note:	E.g. $1.2x-0.03x^2=3$ or $40x-x^2=100$ are acceptable for the 1st M mark
Note:	Give M0 dM0 A0 for (their A)$x^2 = 3 \Rightarrow x = ...$ or their (their A)x^2 + (their k) $= 3 \Rightarrow x = ...$
dM1:	Correct method of solving their quadratic equation to give *at least one solution*
A1:	Interprets their solution in the original context by selecting the larger correct value *and states correct units for their value*. E.g. Accept awrt 37.3 m or $\left(20+\sqrt{300}\right)$ m or $\left(20+10\sqrt{3}\right)$ m
Note:	Condone the use of inequalities for the method marks in part (b)
(c):	
B1:	See scheme
Note:	Give no credit for the following reasons • H (or the height of ball) is negative when $x > 40$ • Bounce of the ball should be considered after hitting the ground • Model will not be true for a different rugby ball • Ball may not be kicked in the same way each time

Q8.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$2x^2 + 4x + 9 = 2(x + b)^2 + c$	B1	This mark is given for writing f(x) in the form $a(x + b)^2 + c$ with $a = 2$
	$2x^2 + 4x + 9 = 2(x + 1)^2 + c$	M1	This mark is given for writing f(x) in the form $a(x + b)^2 + c$ with $a = 2$ and $b = 1$
	$2x^2 + 4x + 9 = 2(x + 1)^2 + 7$	A1	This mark is given for writing f(x) in the form $a(x + b)^2 + c$ with $a = 2$, $b = 1$ and $c = 7$
(b)		B1	This mark is given for a U shaped curve in any position
		B1	This mark is given for a y-intercept shown at (0, 9)
		B1	This mark is given for a minimum shown at (–1, 7)
(c)(i)	$g(x) = 2(x - 2)^2 + 4(x - 2) + 5$	M1	This mark is given for writing g(x) in the form $a(x + b)^2 + c$ and comparing to f(x)
	Translation of $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$	A1	This mark is given for deducing the translation of $y = f(x)$ to $y = g(x)$
(c)(ii)	$h(x) = \dfrac{21}{2(x+1)^2 + 7}$ Maximum value $= \dfrac{21}{7}$ (when $x = -1$)	M1	This mark is given for writing h(x) in the form $\dfrac{21}{a(x+b)^2 + c}$ and finding its maximum value
	$0 < h(x) \leq 3$	A1	This mark is given for finding the correct range of the function h(x)

Q9.

Question	Scheme	Marks	AOs
(a)	$H = ax^2 + bx + c$ and $x=0,\ H=3 \Rightarrow H = ax^2+bx+3$	M1	3.3
	$H = ax^2+bx+3$ and $x=120, H=27 \Rightarrow 27 = 14400a+120b+3$ or $\frac{dH}{dx} = 2ax+b=0$ when $x=90 \Rightarrow 180a+b=0$	M1 A1	3.1b 1.1b
	$H = ax^2+bx+3$ and $x=120, H=27 \Rightarrow 27 = 14400a+120b+3$ **and** $\frac{dH}{dx} = 2ax+b=0$ when $x=90 \Rightarrow 180a+b=0$ $\Rightarrow a = ..., b = ...$	dM1	3.1b
	$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ o.e.	A1	1.1b
		(5)	
(b)(i)	$x=90 \Rightarrow H\left(= -\frac{1}{300}(90)^2 + \frac{3}{5}(90)+3\right) = 30\,\text{m}$	B1	3.4
(b)(ii)	$H=0 \Rightarrow -\frac{1}{300}x^2 + \frac{3}{5}x+3=0 \Rightarrow x = ...$	M1	3.4
	$x = (-4.868...,)\ 184.868...$ $\Rightarrow x = 185\,(\text{m})$	A1	3.2a
		(3)	
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. • The ground is unlikely to be horizontal • The ball is not a particle so has dimensions/size • The ball is unlikely to travel in a vertical plane (as it will spin) • H is not likely to be a quadratic function in x	B1	3.5b
		(1)	
			(9 marks)
	Notes		

....

(a)

M1: Translates the problem into a suitable model and uses $H = 3$ when $x = 0$ to establish $c = 3$

Condone with $a = \pm 1$ so $H = x^2 + bx + 3$ will score M1 but little else

M1: For a correct attempt at **using one of the two other pieces** of information within a quadratic model **Either** uses $H = 27$ when $x = 120$ (with $c = 3$) to produce a linear equation connecting a and b for the model **Or** differentiates and uses $\frac{dH}{dx} = 0$ when $x = 90$. Alternatives exist here, using the symmetrical nature of the curve, so they could use $x = -\frac{b}{2a}$ at vertex or use point (60, 27) or (180,3).

A1: At least one correct equation connecting a and b. Remember "a" could have been set as negative so an equation such as $27=-14400a+120b+3$ would be correct in these circumstances.

dM1: Fully correct strategy that uses $H=ax^2+bx+3$ with the two other pieces of information in order to establish the values of **both** a **and** b for the model

A1: Correct equation, not just the correct values of a, b and c. Award if seen in part (b)

(b)(i)

B1: Correct height including the units. CAO

(b)(ii)

M1: Uses $H=0$ and attempts to solve for x. Usual rules for quadratics.

A1: Discards the negative solution (may not be seen) and identifies awrt 185 m. Condone lack of units

(c)

B1: Candidate should either refer to an issue with one of the four aspects of how the situation has been modelled or give a situation where the model breaks down

- the ball has been modelled as a particle
- there may be trees (or other hazards) in the way that would affect the motion

Condone answers (where the link to the model is not completely made) such as

- the ball will spin
- ground is not flat

Do not accept answers which refer to the situation after it hits the ground (this isn't what was modelled)

- the ball will bounce after hitting the ground
- it gives a negative height for some values for x

Do not accept answers that do not refer to the model in question, or else give single word vague answers

- the height of tee may have been measured incorrectly
- "friction", "spin", "force" etc
- it does not take into account the weight of the ball
- it depends on how good the golfer is
- the shape of the ball will affect the motion
- you cannot hit a ball the same distance each time you hit it

The method using an alternative form of the equation can be scored in a very similar way.
The first M is for the completed square form of the quadratic showing a maximum at $x=90$

So award M1 for $H=\pm a(x-90)^2+c$ or $H=\pm a(90-x)^2+c$. Condone for this mark an equation with $a=1 \Rightarrow H=(x-90)^2+c$ or $c=3 \Rightarrow H=a(x-90)^2+3$ but will score little else

Alt (a)	$H=a(x+b)^2+c$ and $x=90$ at $H_{max} \Rightarrow H=a(x-90)^2+c$	M1	3.3
	$H=3$ when $x=0 \Rightarrow 3=8100a+c$ **or** $H=27$ when $x=120 \Rightarrow 27=900a+c$	M1 A1	3.1b 1.1b
	$H=3$ when $x=0 \Rightarrow 3=8100a+c$ **and** $H=27$ when $x=120 \Rightarrow 27=900a+c$ $\Rightarrow a=...,c=...$	dM1	3.1b
	$H=-\frac{1}{300}(x-90)^2+30$ o.e	A1	1.1b
		(5)	
(b)	$x=90 \Rightarrow H=0^2+30=30\text{m}$	B1	3.4
		(1)	
	$H=0 \Rightarrow 0=-\frac{1}{300}(x-90)^2+30 \Rightarrow x=...$	M1	3.4
	$\Rightarrow x=185\text{(m)}$	A1	3.2a
		(2)	

Note that $H = -\frac{1}{300}(x-90)^2 + 30$ is equivalent to $H = -\frac{1}{300}(90-x)^2 + 30$

Other versions using symmetry are also correct so please look carefully at all responses

E.g. Using a starting equation of $H = a(x-60)(x-120)+b$ leads to $H = -\frac{1}{300}(x-60)(x-120)+27$

Q10.

Question	Scheme		Marks	AOs
(i)	$16a^2 = 2\sqrt{a} \Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	$16a^2 - 2\sqrt{a} = 0$ $\Rightarrow 2a^{\frac{1}{2}}\left(8a^{\frac{3}{2}} - 1\right) = 0$ $\Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	M1	1.1b
	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	M1	1.1b
	$\Rightarrow a = \frac{1}{4}$	$\Rightarrow a = \frac{1}{4}$	A1	1.1b
	Deduces that $a = 0$ is a solution		B1	2.2a
			(4)	
(ii)	$b^4 + 7b^2 - 18 = 0 \Rightarrow (b^2+9)(b^2-2) = 0$		M1	1.1b
	$b^2 = -9, 2$		A1	1.1b
	$b^2 = k \Rightarrow b = \sqrt{k}, k > 0$		dM1	2.3
	$b = \sqrt{2}$, $-\sqrt{2}$ only		A1	1.1b
			(4)	
				(8 marks)

Notes

(i)

M1: Combines the two algebraic terms to reach $a^{\pm\frac{3}{2}} = C$ or equivalent such as $\left(\sqrt{a}\right)^3 = C$ $(C \neq 0)$

An alternative is via squaring and combining the algebraic terms to reach $a^{\pm 3} = k, k > 0$

E.g. $...a^4 = ...a \Rightarrow a^{\pm 3} = k$ or $...a^4 = ...a \Rightarrow ...a^4 - ...a = 0 \Rightarrow ...a\left(a^3 - ...\right) = 0 \Rightarrow a^3 = ...$

Allow for slips on coefficients.

M1: Undoes the indices correctly for their $a^{\frac{m}{n}} = C$ (So M0 M1 A0 is possible)
You may even see logs used.

A1: $a = \frac{1}{4}$ and no other solutions apart from 0 Accept exact equivalents Eg 0.25

B1: Deduces that $a = 0$ is a solution.

(ii)

M1: Attempts to solve as a quadratic equation in b^2
Accept $\left(b^2 + m\right)\left(b^2 + n\right) = 0$ with $mn = \pm 18$ or solutions via the use of the quadratic formula Also allow candidates to substitute in another variable, say $u = b^2$ and solve for u

A1: Correct solution. Allow for $b^2 = 2$ or $u = 2$ with no incorrect solution given.
Candidates can choose to omit the solution $b^2 = -9$ or $u = -9$ and so may not be seen

dM1: Finds at least one solution from their $b^2 = k \Rightarrow b = \sqrt{k}, k > 0$. Allow $b = 1.414$

A1: $b = \sqrt{2}$, $-\sqrt{2}$ only. The solution asks for real values so if $3i$ is given then score A0

Answers with minimal or no working:

In part **(i)**

- no working, just answer(s) with they can score the B1
- If they square and proceed to the quartic equation $256a^4 = 4a$ oe, and then write down the answers they can have access to all marks.

In part (ii)

- Accept for 4 marks $b^2 = 2 \Rightarrow b = \pm\sqrt{2}$
- No working, no marks.

Q11.

Question	Scheme	Marks	AOs
(a) (i)	Uses $\frac{dy}{dx} = -3$ at $x = 2 \Rightarrow 12a + 60 - 39 = -3$	M1	1.1b
	Solves a correct equation and shows one correct intermediate step $12a + 60 - 39 = -3 \Rightarrow 12a = -24 \Rightarrow a = -2$ *	A1*	2.1
(a) (ii)	Uses the fact that $(2,10)$ lies on C $\quad 10 = 8a + 60 - 78 + b$	M1	3.1a
	Subs $a = -2$ into $10 = 8a + 60 - 78 + b \Rightarrow b = 44$	A1	1.1b
		(4)	
(b)	$\text{f}(x) = -2x^3 + 15x^2 - 39x + 44 \Rightarrow \text{f}'(x) = -6x^2 + 30x - 39$	B1	1.1b
	Attempts to show that $-6x^2 + 30x - 39$ has no roots Eg. calculates $b^2 - 4ac = 30^2 - 4 \times -6 \times -39 = -36$	M1	3.1a
	States that as $\text{f}'(x) \neq 0 \Rightarrow$ hence $\text{f}(x)$ has no turning points *	A1*	2.4
		(3)	
(c)	$-2x^3 + 15x^2 - 39x + 44 \equiv (x-4)(-2x^2 + 7x - 11)$	M1 A1	1.1b 1.1b
		(2)	
(d)	Deduces either intercept. $(0,44)$ or $(20,0)$	B1 ft	1.1b
	Deduces both intercepts $(0,44)$ and $(20,0)$	B1 ft	2.2a
		(2)	
			(11 marks)

Notes

(a)(i)

M1: Attempts to use $\frac{dy}{dx} = -3$ at $x = 2$ to form an equation in a. Condone slips but expect to see two of the powers reduced correctly

A1*: Correct differentiation with one correct intermediate step before $a = -2$

(a)(ii)

M1: Attempts to use the fact that $(2,10)$ lies on C by setting up an equation in a and b with $a = -2$ leading to $b = ...$

A1: $b = 44$

(b)

B1: $f'(x) = -6x^2 + 30x - 39$ oe

M1: Correct attempt to show that "$-6x^2 + 30x - 39$" has no roots.
This could involve an attempt at

- finding the numerical value of $b^2 - 4ac$
- finding the roots of $-6x^2 + 30x - 39$ using the quadratic formula (or their calculator)
- completing the square for $-6x^2 + 30x - 39$

A1*: A fully correct method with reason and conclusion. Eg as $b^2 - 4ac = -36 < 0, f'(x) \neq 0$ meaning that no stationary points exist

(c)

M1: For an attempt at division (seen or implied) Eg $-2x^3 + 15x^2 - 39x + b \equiv (x-4)\left(-2x^2 ... \pm \frac{b}{4}\right)$

A1: $(x-4)(-2x^2 + 7x - 11)$ Sight of the quadratic with no incorrect working seen can score both marks.

(d)

See scheme. You can follow through on their value for b