OCR
RECOGNISING ACHIEVEMENT

Investigations for GCSE Mathematics

For teaching from September 2010

Contents

1 Introduction

The changes to GCSE Mathematics place problem solving skills and functionality at the heart of the qualification. In order for learners to be successful, they should be given mathematical activities which allow them to experience

- working with others
- solving problems
- using technology and other resources
- mathematical modelling
- mathematics outside the classroom environment

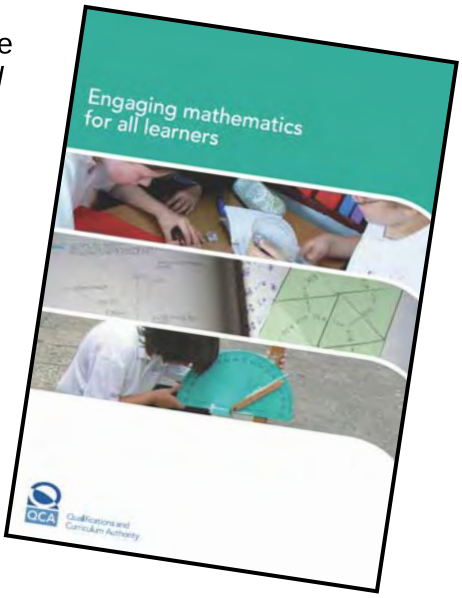
1.1 *Engaging mathematics for all learners*

OCR recommends that teachers read the QCA publication *Engaging mathematics for all learners*, which is available as a PDF from:

http://www.bristol-cyps.org.uk/teaching/secondary/maths/pdf/hub-oct09-engaging-maths.pdf .

The QCA publication gives practical advice for organising rich activities within the curriculum. It includes strategies for devising and using rich activities as well as advice for finding contexts which are mathematically rich. There is also a wide range of examples and case studies, as well as a comprehensive list of resources.

The QCA notes that Ofsted investigations in 2008 found that “in good or outstanding teaching of mathematics, ‘Non-routine problems, open-ended tasks and investigations are used often by all pupils to develop the broader mathematical skills of problem solving, reasoning and generalising.’” (QCA, *Engaging mathematics for all learners*).



This booklet of investigations is intended to provide tasks that can be used in some of the ways described in *Engaging mathematics for all learners* – and they particularly encourage the development of skills with using ICT, maximising and minimising, and changing variables one at a time.

Another suggested activity type in the QCA publication is exploring examination or textbook questions. This is not from the perspective simply of following the method intended, rather to ask open questions such as “How many ways can you find to solve…?” The idea is to change the emphasis of the question to finding as many methods as possible and to appreciate the interconnectedness of concepts.

A further OCR support publication, ‘Practice Questions for GCSE Mathematics’ is ideal for this activity and can be accessed on any of the GCSE Mathematics specifications’ pages.

1.2 OCR's AO3 Problem Solving Guide

OCR's AO3 Problem Solving Guide explains the difference between AO2 and AO3 in the new GCSE Mathematics specifications. It also gives advice about question techniques that can be used to help learners develop thinking skills and a deeper understanding. The guide also provides ten annotated tasks and investigations that can be used as activities to help develop learners' abilities with regards to problem solving and strategy. Depending on your own experience in using open-ended and richer tasks, we would recommend you read and use the activities in the AO3 Problem Solving Guide first before you use the tasks in this booklet.

1.3 OCR & The School Mathematics Project – Support for Problem Solving

A further set of tasks has been carefully developed and trialled by The School Mathematics Project in partnership with OCR. They are not practice exam questions but are instead tasks to help learners develop the mental flexibility that they need for the new elements in the exams.

Like the AO3 Guide, the SMP Problem Solving Tasks Pack is **free to all centres**, irrespective of which specification you are teaching, and available on the OCR website.

1.4 About these investigations

These investigations have all been drawn from OCR's bank of coursework assessments that focussed on *Using and Applying Mathematics*. The coursework element was removed from GCSE Mathematics assessments in September 2007. However, many teachers have told us they would still like to have the investigations available to use, without the pressure of the work forming part of each learner's GCSE grade.

We hope you find these tasks interesting, useful and rewarding.

2 The investigations

Dodgy Dice

Pawel and Becky have invented a game.

The game is played on a square board, with squares numbered from one to one hundred.

GO	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
	21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
	31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
	41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
	51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
	61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
	71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
	81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
	91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**

- The dice Pawel and Becky use only has the numbers 4 and 7 on it.
- They start on **GO** and take it in turns to throw the dice. They move forward the number of squares shown on the dice. The first to get to square 100 is the winner.

Pawel soon notices that neither of them can ever land on square 1, square 2 or square 3.

Becky notices that there are some other squares that are never landed on.

1 What is the next square after square 3 that cannot be landed on?

2 Which square has the biggest number on it that cannot be landed on?

Becky decides to change the numbers on the dice to 3s and 5s.

3 Which squares cannot be landed on now?

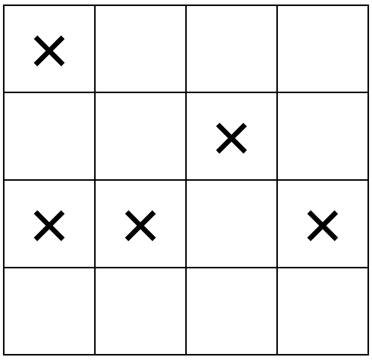
4 Investigate further.

Winning Lines

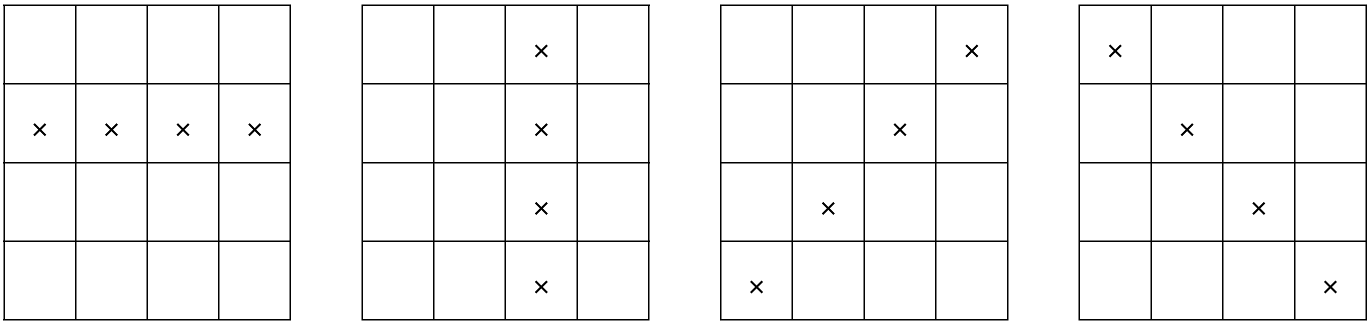
Jemma and Roberto are playing a game on a 4 by 4 square grid.

They take it in turns to place a cross in an empty square on the grid.

The winner is the first player to complete a line of four crosses.



In this game, winning lines are horizontal, vertical or diagonal.



1 How many different horizontal winning lines can be made on the 4 by 4 grid?

2 How many different winning lines can be made <u>in total</u> on the 4 by 4 grid?

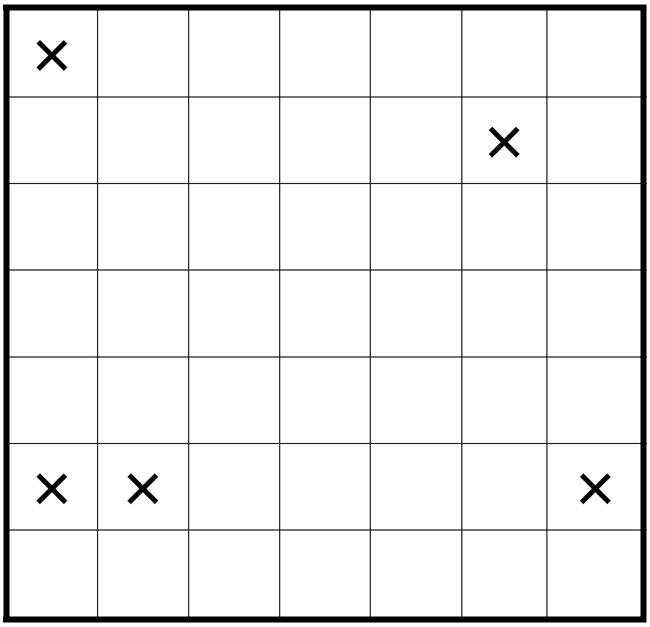
3 Investigate further.

Winning Lines 2

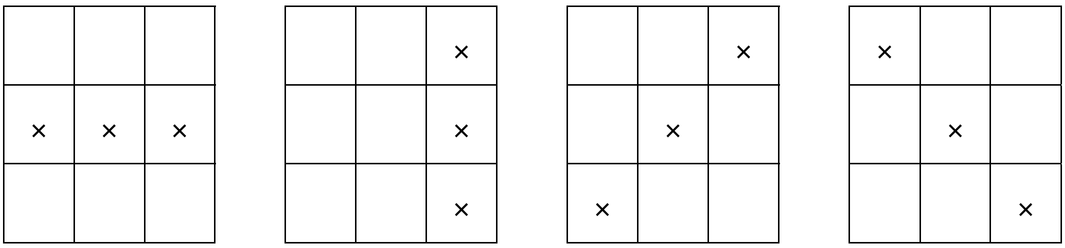
Jemma and Roberto are playing a game on a 7 by 7 square grid.

They take it in turns to place a cross in an empty square on the grid.

The winner is the first player to complete a line of three crosses.



In this game, winning lines are horizontal, vertical or diagonal.



1 How many different horizontal winning lines are possible on the 7 by 7 grid?

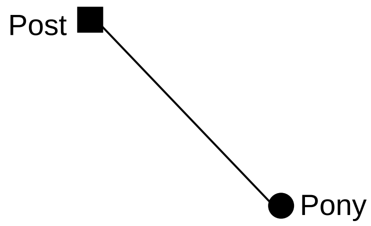
2 Extend your investigation, making clear the rules and the methods that you use.

Grazing

You may wish to use 5 mm or 1 cm squared paper to help you with this activity.

1 Isaac keeps a pony in a large grassy field.

There is a fence all round the field.
The pony is tied to a post by a rope 4 m long.
The post is in the middle of the field.



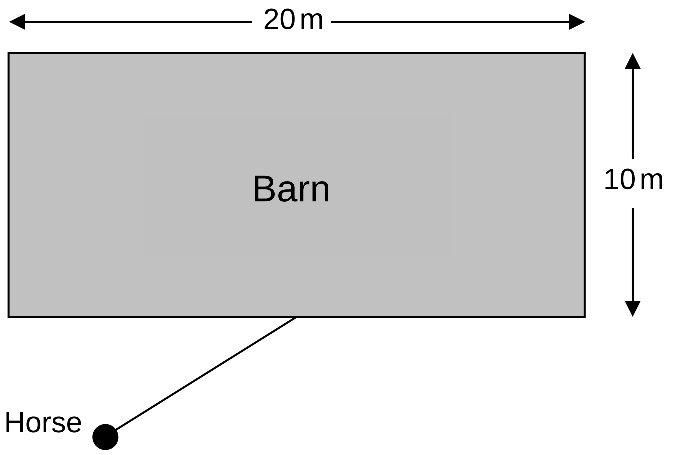
(a) What shape is made in the grass when the pony has eaten all of the grass it can reach?
(b) What happens to this shape if the post is moved nearer to the fence around the field?

2 The pony needs an area of 100 m^2 of grass each day.

(a) What area of grass does the pony have when the rope is 4 m long?
(b) What length should the rope be so that the pony just has the area of grass it needs?

3 Shamina keeps a horse in a field that has a big barn in the middle of it.

The barn is 20 m long and 10 m wide.
The horse is tied to the outside of the barn, halfway along one of the long sides.

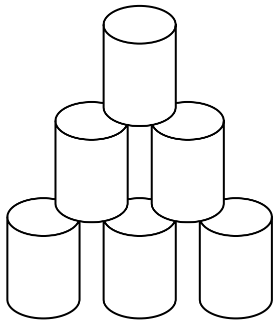


How long should the rope be so that the horse just has 150 m^2 of grass?

4 Investigate further.

Pile ‘em high

Azeem works in a supermarket.
He is building a display of soup tins by stacking them.
The tins are stacked up against a wall
Azeem stacks the tins using a ‘brick band’ to make the display stable.
This means that each tin stands on two others, as shown below.

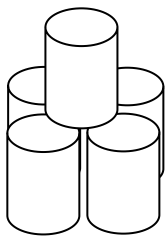


1 How many tins are needed for a stack 4 tins high?

2 Azeem thinks that all stacks more than 4 tins high need an odd number of tins. Is this true?

3 Azeem has 210 tins. How many tins high is the tallest stack he can build?

Azeem is asked to build a free-standing display, which is not against a wall.
An example is shown below.



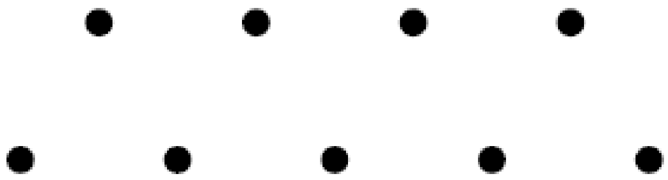
4 Investigate further.

Spotty Roofs

You will need triangular dotty paper for this activity.

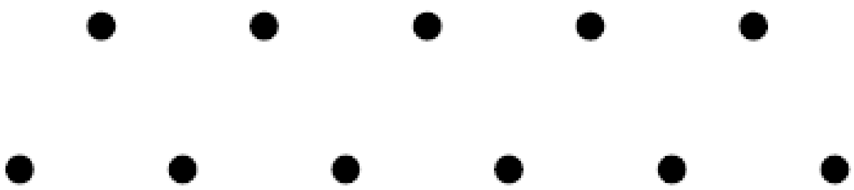
A roof is made out of spots in the shape of a trapezium.

1 This is described as a 5-spot roof.



How many spots are used to make this roof?

2 Here is another roof.

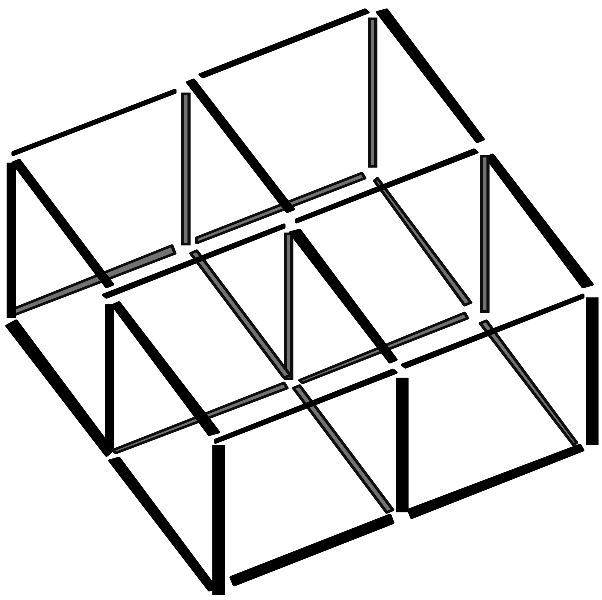


(a) How would this roof be described?

(b) How many spots are used to make it?

3 Investigate further.

Matchstick Models



This Matchstick Model is made using 33 matchsticks.
How could you count them easily and reliably?

Investigate the number of matchsticks needed to make other models.

Opposite Corners

1	**2**	**3**	**4**
5	**6**	**7**	**8**
9	**10**	**11**	**12**
13	**14**	**15**	**16**

The numbers 1 and 16 are one pair of opposites in this 4 by 4 grid.
The numbers 4 and 13 are the other pair of opposites.

$4 \times 13 = 52$
$1 \times 16 = 16$

and $52 - 16 = 36$.

Investigate opposite corners on grids.

Magic E

A 10 by 8 grid is numbered from 1 to 80.
An E shape is placed over the grid.
The first E is shown on the grid below.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
17	**18**	**19**	**20**	**21**	**22**	**23**	**24**
25	**26**	**27**	**28**	**29**	**30**	**31**	**32**
33	**34**	**35**	**36**	**37**	**38**	**39**	**40**
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**
49	**50**	**51**	**52**	**53**	**54**	**55**	**56**
57	**58**	**59**	**60**	**61**	**62**	**63**	**64**
65	**66**	**67**	**68**	**69**	**70**	**71**	**72**
73	**74**	**75**	**76**	**77**	**78**	**79**	**80**

All the numbers in the E are added together.

1 What is the total of the numbers in the first E?

2 The E is moved one square to the right to create the second E.

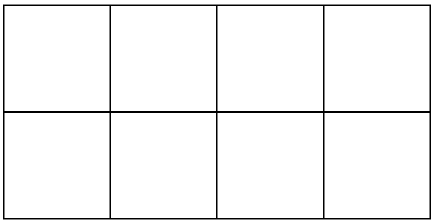
What is the total of the numbers in the second E?

3 Investigate the total of the numbers in the E for any position on the grid.

4 Investigate further.

How Many?

1 The diagram below shows a rectangle which is 2 cm wide and 4 cm long.

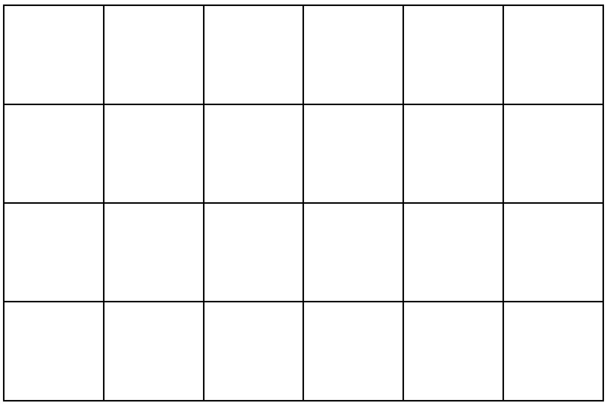


Bella and Juan are arguing about the number of squares in the diagram. Bella says there are 8 squares but Juan says there are 11.

Who is correct, Bella or Juan?

2 Investigate the number of squares in rectangles of width 2 cm.

3 How many squares are there in the diagram below?



4 Extend your investigation, making clear the rules and methods you use.

Anyone for T?

A 10 by 10 grid is numbered from 1 to 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The grid has a letter T placed on it, as shown on the diagram.
This T has a horizontal bar of length 3 and a vertical bar of length 2.

The numbers at each end of the horizontal bar are added. This answer is then subtracted from the number at the bottom of the vertical bar. The result is called the T value.

This is the working for T_1

$1 + 3 = 4$

So T_1 is $22 - 4 =$ **18**

1 Work out T_2 and T_3

2 Investigate T values as the T moves around the grid.

3 Extend your investigation, making clear the rules and methods you use.

Anyone for T 2?

A 10 by 10 grid is numbered from 1 to 100.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**

The grid has a letter T placed on it, as shown on the diagram.
This T has a horizontal bar of length 3 and a vertical bar of length 2.

The number at the bottom of the vertical bar is squared and the numbers at each end of the horizontal bar are multiplied. One answer is then subtracted from the other. This result is called the T value.

This is the working for T_{13}

$34^2 = 1156$

$13 \times 15 = 195$

So T_{13} is $1156 - 195 =$ **961**

1 Investigate T values as the T moves around the grid.

2 Extend your investigation, making clear the rules and methods you use.

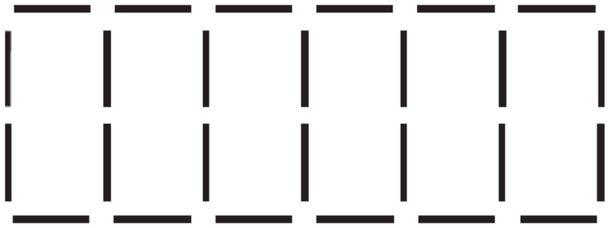
Matchstick Patterns

Gemma is making patterns with matchsticks. Here are some of the patterns she has made.

Picture A

Picture B

Picture C



Picture A shows a pattern 1 matchstick wide.
Picture B shows a pattern 3 matchsticks wide.
Picture C shows a pattern 6 matchsticks wide.

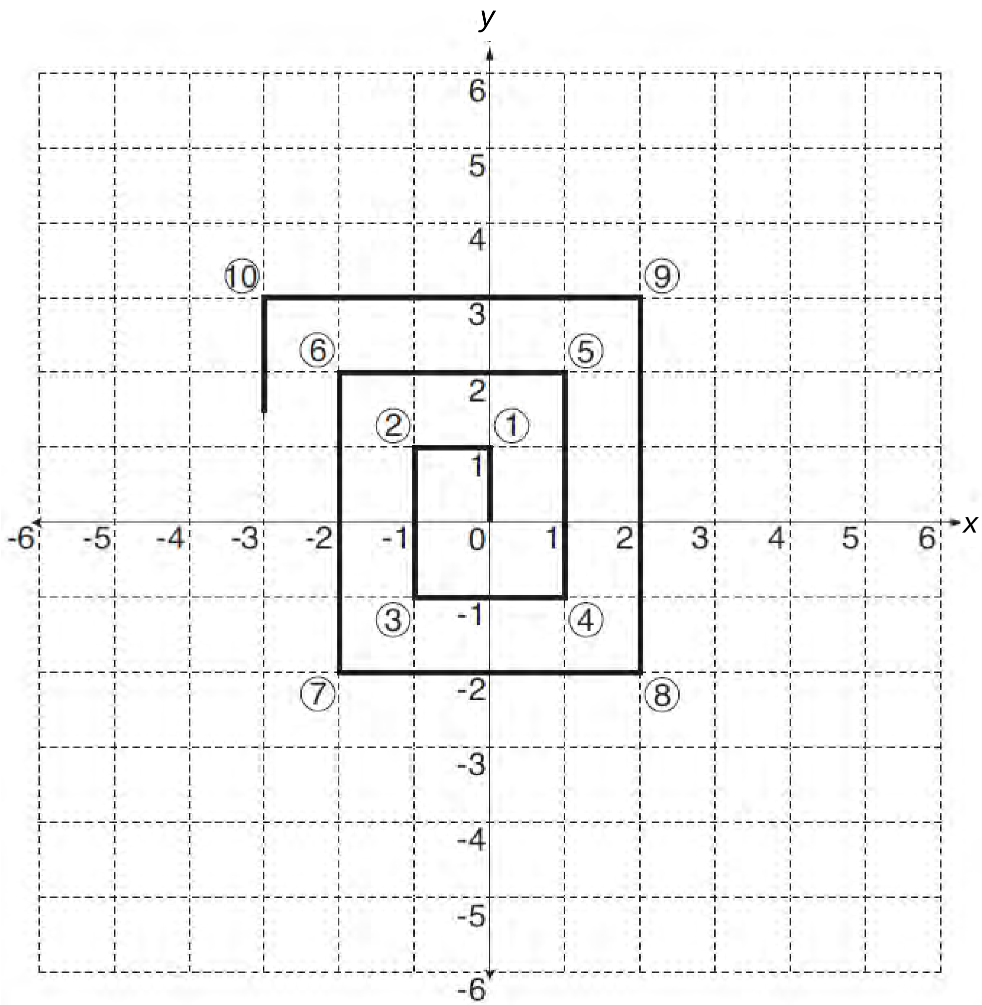
1 Gemma used 6 matchsticks to make the pattern in Picture A.
How many matchsticks are used to make each of the patterns shown in Pictures B and C?

2 Gemma's patterns are all 2 matchsticks high.
Investigate the link between the width of each pattern and the number of matchsticks used to make it.

3 Extend your investigation to other patterns made with matchsticks, making clear the rules and methods that you use.

Spiral Bound

A spiral is drawn on a square grid. Part of the spiral is shown on this diagram.



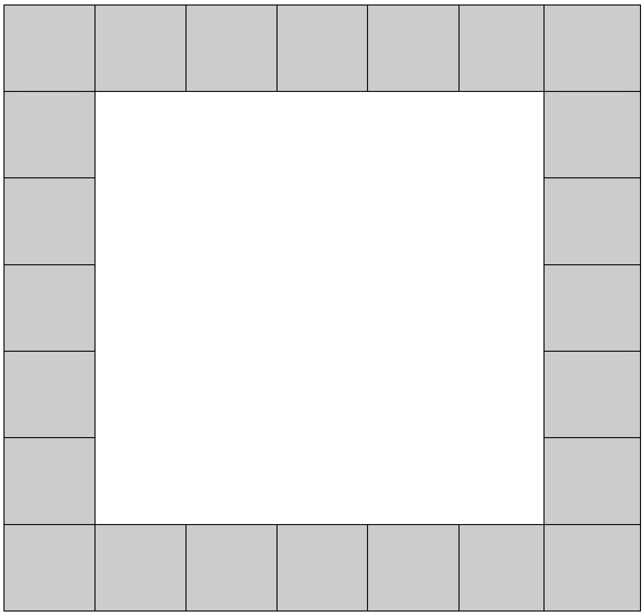
The spiral starts at (0, 0) and its corners are labelled ①, ②, ③, … .
The length of the spiral from the origin (0, 0) to corner ⑤ at (1, 2) is 9 units.

1 What is the length of the spiral from the origin (0, 0) to corner ⑩ at (–3, 3)?

2 Investigate the length of the spiral from the origin to different corners.

3 Extend your investigation, making clear the rules and methods that you use.

Mirrrors



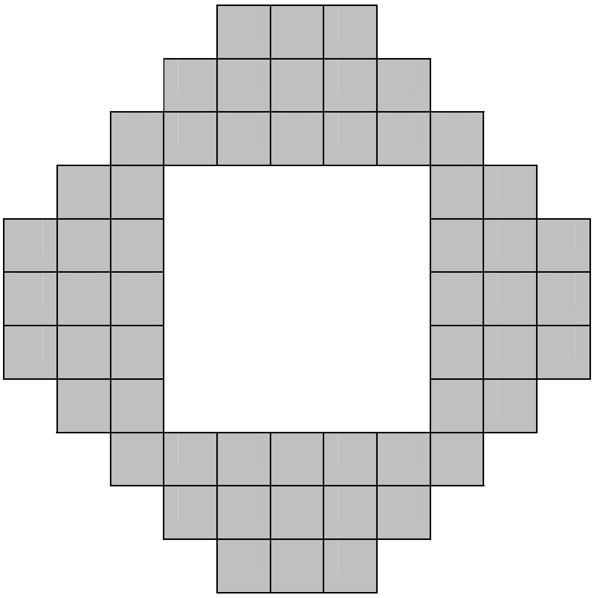
Sasha uses tiles to make borders for square mirrors. The picture shows a design for a 5 by 5 mirror with a border of 1 by 1 tiles.

1 What is the total number of 1 by 1 tiles used to make a border for
  (i) a 5 by 5 mirror,
  (ii) a 7 by 7 mirror?

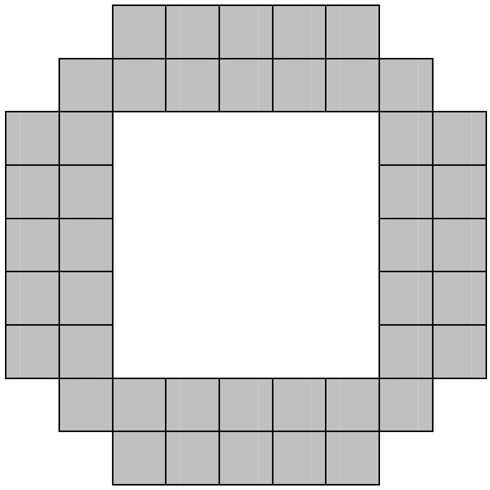
2 Investigate the total number of 1 by 1 tiles that are needed to make borders for different sized square mirrors.

3 Extend your investigation to other mirrors with borders, making clear the rules and methods that you use.

Mirrrors 2



3 row design



2 row design

Sasha uses tiles to make borders for square mirrors.
The picture shows two possible designs for a 5 by 5 mirror surrounded with 1 by 1 tiles.

1 Choose either the 3 row design or the 2 row design.

(a) What is the total number of 1 by 1 tiles used to make a border for

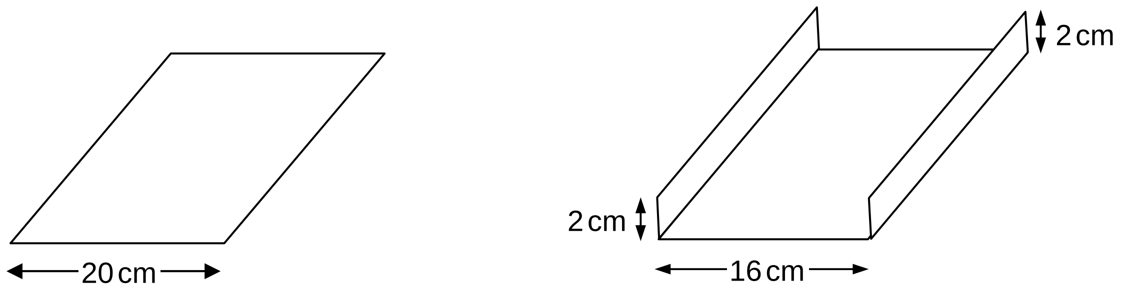
- **(i)** a 5 by 5 mirror,
- **(ii)** a 7 by 7 mirror?

(b) Investigate the total number of 1 by 1 tiles that are needed to make borders for different sized square mirrors.

2 Investigate ‘borders’ of objects, making clear the rules and methods that you use.

Guttering

Guttering is made from a piece of rectangular plastic 20 cm wide. The plastic is bent or shaped to produce guttering of different cross-sections that will allow water to flow along it.

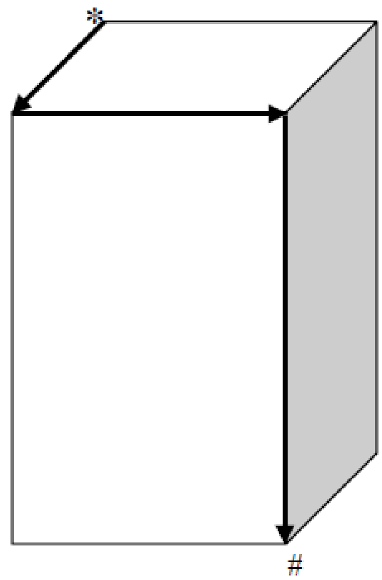


The above diagram shows one way some guttering can be made from the plastic.
The cross-section of the guttering is a rectangle shape, measuring 16 cm by 2 cm.
So the area of the cross-section is 32 cm^2

Investigate which shape guttering will give the maximum flow of water.

Routes on Polyhedra

The diagram below is the sketch of a cuboid.
(The base is horizontal.)



The top corner of the cube is marked * and the bottom corner is marked #.
The arrows show a route from the top corner to the bottom corner.

The rules for finding a route are:

- Start at the corner marked *.
- Move only along the edges of the cuboid.
- Only move along one edge **once** in any one route.
- Never move upwards.
- Stop at the corner marked #.

1 Investigate the number of different routes from the corner marked * to the corner marked #.

2 Investigate the number of different routes from a top corner to a bottom corner for solid shapes.

Three Shorter Investigations

1 Noughts and Crosses

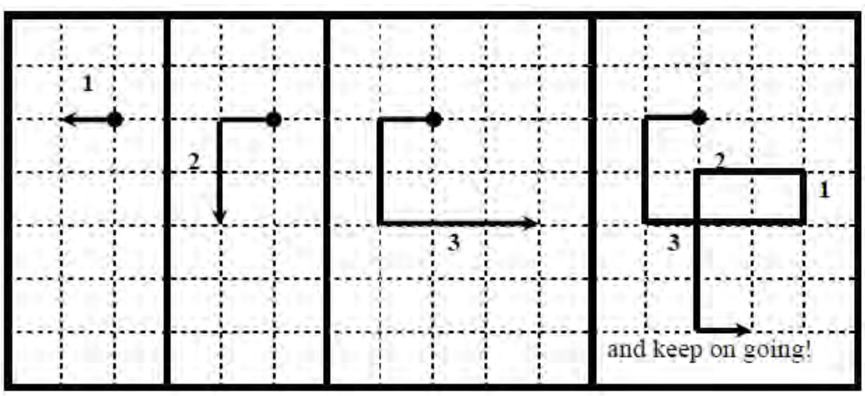
x	O	O
O	O	x
x	O	x

How many winning lines are there in a normal game of noughts and crosses?

Investigate other possibilities.

2 Spirals

This is how you create the spiral “1, 2, 3”.



Investigate spirals.

3 Area and Coordinates

Find a connection between the areas of triangles like ABO and the coordinates of the two vertices A and B. The third vertex, O, is always at the origin, (0, 0).