20/02/17

Es 2

$$f(x) = \begin{cases} \dfrac{\sin(x^3)}{xy} & xy \neq 0 \\ 0 & xy = 0 \Rightarrow x=0 \vee y=0 \end{cases}$$

$$\frac{\partial f}{\partial x}(\underline{0}) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = 0$$

$$\frac{\partial f}{\partial y}(\underline{0}) = \lim_{h\to 0} \frac{f(0,h)-f(0,0)}{h} = 0$$

f continua in $\underline{x}=\underline{0}$? OK

$$\lim_{(x,y)\to(0,0)} f(x,y) \sim \frac{\sin(\rho^3\cos^3\theta)}{\rho^2\sin\theta\cos\theta} \sim \frac{\rho^{\cancel{3}}\cos^3\theta}{\cancel{\rho^2}\sin\theta\cos\theta} \to 0$$

f non continua in tutto il suo dominio ma è cont. in $U((0,0))$ $\Rightarrow$ è risolto il dubbio!

$$\underline{v} = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}$$

$$\frac{\partial f}{\partial \underline{v}}(\underline{0}) = \lim_{h\to 0} \frac{f\left(\frac{\sqrt{3}}{2}h, \frac{1}{2}h\right) - f(0,0)}{h} \sim$$

$$\sim \frac{\sin\left(\frac{3\sqrt{3}}{8}h^3\right)}{\frac{\sqrt{3}}{4}h^2} \cdot \frac{1}{h} \sim$$

$$\sim \frac{3\cancel{\sqrt{3}}}{8}\cancel{h^3} \cdot \frac{4}{\cancel{\sqrt{3}}\,\cancel{h^3}} \longrightarrow \frac{3}{2}$$

Non vale: $\dfrac{\partial f}{\partial \underline{v}}(\underline{0}) = \dfrac{\sqrt{3}}{2}\dfrac{\partial f}{\partial x}(\underline{0}) + \dfrac{1}{2}\dfrac{\partial f}{\partial y}(\underline{0}) \Rightarrow$

$\Rightarrow f$ non differenziabile

19/04/17

es. 5

$$f(x,y) = \frac{y^2}{1+x^2}$$

$$f(1,1) = \frac{1}{2} \qquad P\left(1, 1, \frac{1}{2}\right)$$

$? = T_P f$

$$\frac{\partial f}{\partial x} = -\frac{y^2}{(1+x^2)^2} \cdot 2x$$

$$\frac{\partial f}{\partial y} = \frac{2y}{1+x^2} \qquad \nabla f(1,1) = \begin{pmatrix} -1/2 \\ 1 \end{pmatrix}$$

$T_P f$:

$$z - \frac{1}{2} = \nabla f(1,1) \cdot \begin{pmatrix} x-1 \\ y-1 \end{pmatrix}$$

$$z - 1/2 = -\frac{1}{2}(x-1) + y - 1$$

$$\underline{v} = \begin{pmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{pmatrix} \qquad ? = \frac{\partial f}{\partial \underline{v}}(1,1) = D_{\underline{v}} f(1,1)$$

$$\frac{\partial f}{\partial \underline{v}}(\underline{x}_0) = \lim_{h \to 0} \frac{f(x_0 + h v_x, y_0 + h v_y) - f(x_0, y_0)}{h}$$

$$\underline{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} \qquad |\underline{v}| = 1$$

Teor (diff. totale)

f è diff. in $\underline{x}_0$ $\Rightarrow$ $\frac{\partial f}{\partial \underline{v}}(x_0) = v_x \frac{\partial f}{\partial x}(\underline{x}_0) + v_y \cdot \frac{\partial f}{\partial y}(\underline{x}_0)$

Nell'esercizio: $\frac{\partial f}{\partial \underline{v}}(1,1) = \frac{2}{\sqrt{5}}\left(-\frac{1}{2}\right) + \frac{1}{\sqrt{5}} \cdot 1 = 0$

App 4/2019

$$f(\underline{x}) = \begin{cases} \dfrac{x^3 e^y - y\sin(x^3)}{x^2+y^2} & \underline{x} \neq \underline{0} \\ 0 & \underline{x} = \underline{0} \end{cases}$$

f cont. in $\underline{x} = \underline{0}$?

$$\lim_{\underline{x}\to\underline{0}} f(\underline{x}) \underset{\rho\to 0}{\sim} \frac{\rho^3\cos^3\vartheta\, e^{\rho\sin\vartheta} - \rho\sin\vartheta \sin(\rho^3\cos^3\vartheta)}{\rho^2} \sim$$

$$\sim \frac{\rho^{\cancel{3}}\cos^3\vartheta - \rho^{\cancel{4}\,2}\sin\vartheta\sin(\cos^3\vartheta)}{\cancel{\rho^2}} \xrightarrow[\rho\to 0]{} 0 \qquad f \text{ cont in } \underline{x} = \underline{0}$$

$$\frac{\partial f}{\partial x}(\underline{0}) = \lim_{h\to 0} \frac{h^3}{h^2}\cdot\frac{1}{h} = 1 \qquad f \text{ derivabile in } \underline{x} = \underline{0}$$

$$\frac{\partial f}{\partial y}(\underline{0}) = \lim_{h\to 0} 0 = 0$$

$$\underline{v} = \begin{pmatrix} \cos\vartheta \\ \sin\vartheta \end{pmatrix}$$

$$\frac{\partial f}{\partial \underline{v}}(\underline{0}) = \lim_{h\to 0} \frac{f(h\cos\vartheta, h\sin\vartheta) - f(0,0)}{h} \sim$$ (analogamente al calcolo della continuità)

$$\sim \frac{\cancel{h}\cos^3\vartheta}{\cancel{h}} \to \cos^3\vartheta$$

Se f fosse differenziabile in $\underline{x} = \underline{0}$:

$$\frac{\partial f}{\partial \underline{v}}(\underline{0}) = \cos\vartheta\cdot 1 + \sin\vartheta\cdot 0 = \cos\vartheta$$

(non vale il teor. del diff totale!) $\Rightarrow$ f non diff in $\underline{x} = \underline{0}$

Teor (funzione implicita del Dini)

$f : \mathrm{Dom} f \subseteq \mathbb{R}^2 \to \mathbb{R}$

$f(x;y) = 0$

$(x_0; y_0) \in \mathrm{Dom} f \ : \ f(x_0; y_0) = 0$

Se $\frac{\partial f}{\partial y}(x_0; y_0) \neq 0 \Rightarrow \exists$ loc. in $U(x_0)$ una funzione implicita $y = y(x)$ definita da $f(x; y(x)) = 0$

$$\boxed{y'(x_0) = -\frac{\frac{\partial f}{\partial x}(x_0; y_0)}{\frac{\partial f}{\partial y}(x_0; y_0)}}$$

es $f(x;y) = x^2 - y = 0$ definisce la funzione $y = x^2$

es $f(x;y) = e^{xy} - \ln y = 0$ $\quad y = y(x)$?

18/01/19
per giov
es 1

$$f(x;y) = y^2 - 4x^2 - \sin(2x - y) = 0$$

$\exists (1;2)$

$$\frac{\partial f}{\partial y}(1;2) = \left(2y + \cos(2x - y)\right)\Big|_{(1;2)} = 4 + \cos(0) = 5 \neq 0$$

Teor delle f. implicita : $\exists\, y = y(x)$ f. implicita in $U(1)$ $(x_0 = 1)$

$T_{x_0} y$: $y - \underset{2}{\underset{\shortparallel}{y(1)}} = y'(1)(x-1)$

$$y'(1) = \frac{-\frac{\partial f}{\partial x}(P)}{\frac{\partial f}{\partial y}(P)} = -\frac{(-10)}{5} = 2$$

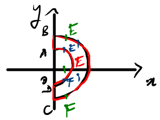
$$\frac{\partial f}{\partial x}(1;2) = \left(-8x - 2\cos(2x - y)\right)\Big|_{\underline{x} = (1,2)} = -8 - 2\cos(0) = -10$$

$T_{x_0} y$: $y - 2 = 2(x-1)$

18/01/19
es. 5

$f(x,y) = x^3 + y^2$

$E = \{ \underline{x} \in \mathbb{R}^2 : x \geq 0, \ 1 \leq |x| \leq 2 \}$



f pari risp. a y
E compatto

E compatto $\Rightarrow$ f ammette max e min su E
Teor di Weierstrass

$\frac{\partial f}{\partial x} = 3x^2 \qquad \frac{\partial f}{\partial y} = 2y$

$\underline{\nabla} f = \underline{0} \quad \begin{cases} x = 0 \\ y = 0 \end{cases} \quad (0,0) \notin E$

non ci sono estremi liberi in E

Devo cercare max e min su ∂E

Su AB e CD: $f(0,y) = g_1(y) = y^2$

$g_1'(y) = 2y$

g_1'
g_1

$f(C) = 4 = f(B)$
$f(D) = 1 = f(A)$

Su $\widehat{BC}$: $x^2 + y^2 = 4 \Rightarrow y^2 = 4 - x^2$

$f_{|\widehat{BC}}(x,y) = g_2(x) = x^3 + 4 - x^2$

$g_2'(x) = 3x^2 - 2x \geq 0 \Rightarrow x \leq 0 \vee x \geq \frac{2}{3}$

g_2'

$E\left(\frac{2}{3}, \frac{4\sqrt{2}}{3}\right)$

$F\left(+\frac{2}{3}, -\frac{4\sqrt{2}}{3}\right)$

$g_2(0) = f(0, \pm 2) = f(B) = f(C) = 4$

$f(E) = f(F) = \frac{8}{27} + \frac{32}{9} = \frac{104}{27} < 4$

$g_2(2) = f(2,0) = 8$ CORREZIONE $*$

Su $\widehat{AD}$: $y^2 = 1 - x^2$

$f_{|\widehat{AD}}(x,y) = g_3(x) = x^3 + 1 - x^2$

$g_3'(x) = 3x^2 - 2x$

$E'\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$

Analogamente al caso di $\widehat{BC}$

$F'\left(\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$

$g_3(0) = f(A) = f(D) = 1$

$g_3\left(\frac{2}{3}\right) = f(E') = f(F') = \frac{8}{27} + \frac{5}{9} = \frac{8+15}{27} = \frac{23}{27} < 1$

CORREZIONE $*$

Confrontando tutti i punti critici

$g_3(1) = f(1,0) = 1$

$(2,0)$ è il punto di max assoluto CORREZIONE $*$

E', F' sono punti di min assoluto

20/02/17

es. 4

$$\begin{cases} y'' - 4y' + 13y = 13 \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$$

$$y = y_0 + \bar{y}$$

Polinomio associato: $\lambda^2 - 4\lambda + 13 = 0$

$$\lambda = 2 \pm \sqrt{4-13} \qquad \Delta < 0$$

$$\lambda = 2 \pm 3i$$

$$y_0 = A\cancel{e^{(2-3i)x}} + B\cancel{e^{(2+3i)x}} = e^{2x}\left(A\cos(3x) + B\sin(3x)\right)$$

$\bar{y} = \alpha$: $\quad 13\alpha = 13 \Rightarrow \alpha = 1$

Int. generale: $\quad y = e^{2x}\left(A\cos(3x) + B\sin(3x)\right) + 1$

$y(0) = 0$: $\quad 0 = A + 1 \Rightarrow A = -1$

$$y = e^{2x}\left(-\cos(3x) + B\sin(3x)\right) + 1$$

$y'(0) = 1$: $\quad y' = 2e^{2x}\left(-\cos(3x) + B\sin(3x)\right) + e^{2x}\left(3\sin(3x) + 3B\cos(3x)\right)$

$$y'(0) = -2 + 3B = 1 \Rightarrow B = +1$$

$$y = e^{2x}\left(-\cos(3x) + \sin(3x)\right) + 1$$